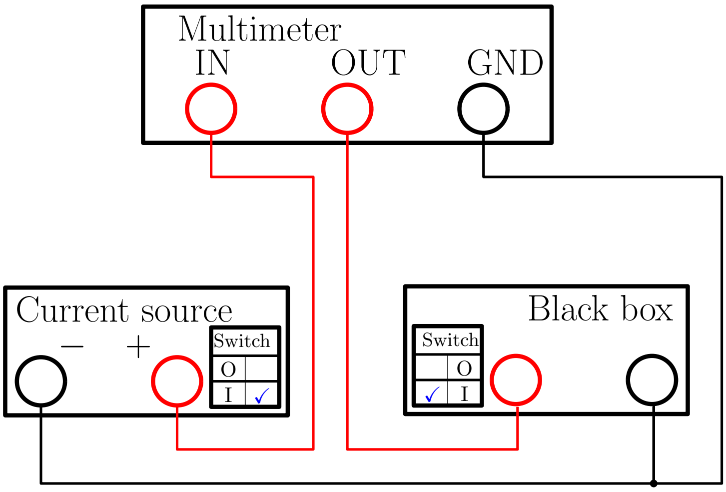
Problem E2. Nonlinear Black Box *(10 points)*

Part A. Circuit without inductance *(7 points)*

It is possible to make all the measurements needed for this problem with a single circuit as shown in the figure. While the current source is switched on, we are charging the capacitor in the black box, until the current $I(V_{\max})$ through the nonlinear element equals to the output current I_0 of the current source. $V_{\max} = 540 \pm 40\,\mathrm{mVs}$ varies from one experimental setup to another. When the current source is switched off or disconnected, the capacitor will discharge through the nonlinear element.



i. (1 pt) During charging of the capacitor from $V = 0$ to $V = V_{\max}$ we note that the output of the current source is constant ($I_0 = 6.0\,\mathrm{mA}$) close to the precision of the multimeter.

ii. (1.2 pts) Using the definition of differential capacitance, we can calculate the current through the capacitor in the black box from the time derivative of the voltage on the black box.

$$I_c = \frac{dQ}{dt} = \frac{dQ}{dV}\frac{dV}{dt} = C(V)\dot{V}$$

There are several ways to determine the capacitance used in the black box based on chosen voltage.

- When the voltage on the black box is close to zero, the current through the nonlinear element is also close to zero, because $I(V = 0) = 0$. After switching the current source on, most of the input current I_0 will at first go through the capacitor.

  $$C_0 = I_0/\dot{V}_{\uparrow}(V = 0)$$

  This can be measured more precisely after first reversing the polarity of the current source and charging the capacitor backwards, because the multimeter does not display derivatives when they change sharply (as in few moments after switching the current source on).

  Example measurements taken this way follow.

$\dot{V}_{\uparrow}(0)$ (mV/s)	3.51	3.32	3.55
C_0 (F)	1.71	1.81	1.69

$$C_0 = 1.74\,\mathrm{F}$$

- When the voltage on the black box is $V_{\max}$, the current through the nonlinear element is I_0. Switching the current source off, we will have the capacitor discharging with the same current.

  $$C_0 = -I_0/\dot{V}_{\downarrow}(V = V_{\max})$$

- We can also measure the capacitance for any intermediate voltage as in **A-iv**.

iii. (2.2 pts) If we neglect the nonlinearity of the capacitor, there are (at least) two ways to obtain the current–voltage characteristic of the nonlinear element in the black box.

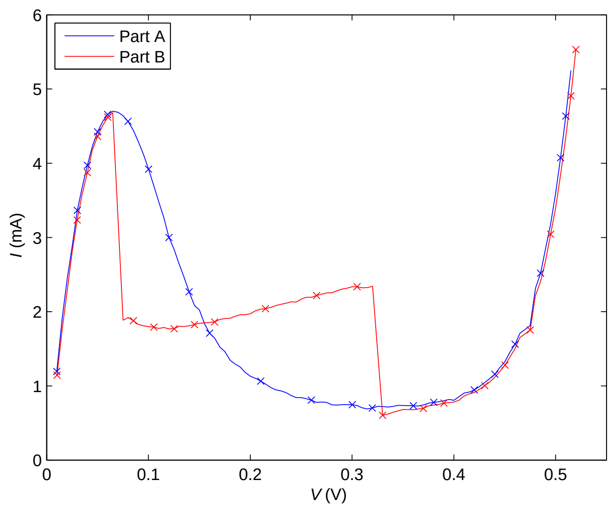
- Applying Kirchhoff's I law to the charging capacitor,

  $$I(V) = I_c - C_0\dot{V}_{\uparrow}(V).$$

  An $I(V)$ characteristic obtained by charging the capacitor is shown on the following figure.

- Applying Kirchhoff I law to the discharging capacitor,

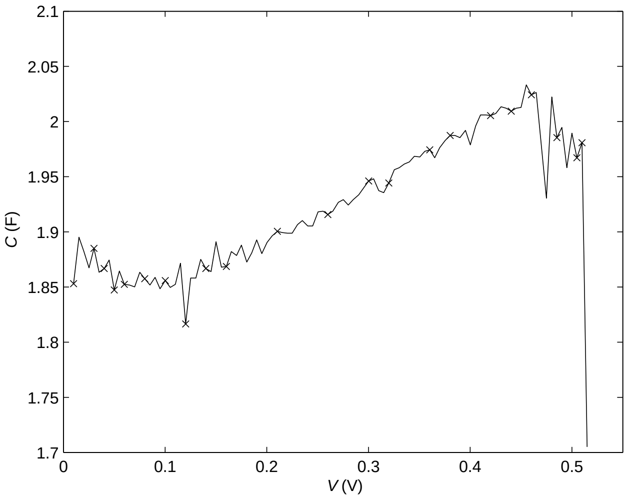
  $$I(V) = -C_0\dot{V}_{\downarrow}(V).$$



iv. (2.6 pts) In order to obtain the differential capacitance, we solve a system of linear equations by eliminating $I(V)$:

$$\begin{cases} I_0 = \dot{V}_{\uparrow}C(V) + I(V) \\ I(V) = -\dot{V}_{\downarrow}C(V); \end{cases} \implies C(V) = \frac{I_0}{\dot{V}_{\uparrow} - \dot{V}_{\downarrow}}.$$

Therefore we need to take measurements during both charging and discharging the capacitor in the black box at the same voltages. A graph of measurement results follows.



Part B. Circuit with inductance *(3 points)*

Measuring and plotting the current–voltage characteristic of the nonlinear element in the same way as in part **A-iii**, we obtain a graph that differs only in the negative differential resistance ($I'(V) < 0$) region, in our case $70\,\mathrm{mV} < V < 330\,\mathrm{mV}$. This is the region where, when we look at small-signal oscillations, the nonlinear element behaves as a negative-valued Ohmic resistance. After enabling the inductance we have a LC circuit whose oscillations are amplified (instead of being dampened) by the negative differential resistance. Because the resonant frequency $\omega = \sqrt{\frac{1}{LC_p}} \sim 30\,\mathrm{MHz}$ (with C_p being the capacitance of the nonlinear element) is high, we actually measure the average current through the nonlinear element, while the real current oscillates all over the region of negative differential resistance.