Problem E1. The magnetic permeability of water *(10 points)*

Part A. Qualitative shape of the water surface *(1 points)*

Observing reflections from the water surface (in particular, those of straight lines, such as the edge of a sheet of paper), it is easy to see that the profile has one minimum and has a relatively flat bottom, ie. the correct answer is "Option D" (full marks are given also for Option B). This profile implies that water is pushed away from the magnet, which means $\mu < 1$ (recall that ferromagnets with $\mu > 1$ are pulled).

Part B. Exact shape of the water surface *(7 points)*

i. (1.6 pts) The height of the spot on the screen y is tabulated below as a function of the horizontal position x of the caliper. Note that the values of y in millimetres can be rounded to integers (this series of measurements aimed as high as possible precision).

x (mm)	10	15	20	25	30	32	34	36
y (mm)	11.5	15.6	19.8	24.3	30.2	33.2	37.2	40.5
x (mm)	38	40	42	44	46	48	50	52
y (mm)	42.2	41.4	40.3	40.3	40.8	42	43.2	44.4
x (mm)	54	56	58	60	62	64	66	68
y (mm)	45.3	45.8	45.4	44.4	43.6	46.2	50	53.6
x (mm)	70	72	74	76	78	80	85	90
y (mm)	56.7	59.5	61.6	63.5	65.3	67	70.9	74.9

ii. (0.7 pts)

On this graph, the data of to two different water levels are depicted; blue curve corresponds to a water depth of *ca 2 mm* (data given in the table above); the violet one — to 1 mm.

iii. (0.5 pts) If the water surface were flat, the dependence of x on y would be linear, and the tangent of the angle α_0 would be given by $\tan\alpha_0 = \frac{\Delta y}{\Delta x}$, where Δx is a horizontal displacement of the pointer, and Δy — the respective displacement of the spot height. For the extreme positions of the pointer, the beam hits the water surface so far from the magnet that there, the surface is essentially unperturbed; connecting the respective points on the graph, we obtain a line corresponding to a flat water surface — the red line. Using these two extreme data points we can also easily calculate the angle $\alpha_0 = \arctan\frac{74.9-11.5}{90-10} \approx 38°$.

iv. (1.4 pts) For faster calculations, $y - y_0 - (x - x_0)\tan\alpha_0$ (appearing in the formula given) can be read from the previous graph as the distance between red and blue line; the red line is given by equation $y_r = y_0 + (x - x_0)\tan\alpha_0$. One can also precalculate $\frac{1}{2}\cos^2\alpha_0 \approx 0.31$. The calculations lead to the following table (with $z = \tan\beta \cdot 10^5$; as mentioned above, during the competition, lesser precision with two significant numbers is sufficient).

x (mm)	10	15	20	25	30	32	34	36
z	0	10	27	66	204	303	473	591
x (mm)	38	40	42	44	46	48	50	52
z	597	428	239	128	53	26	0	-26
x (mm)	54	56	58	60	62	64	66	68
z	-72	-145	-278	-449	-606	-536	-388	-254
x (mm)	70	72	74	76	78	80	85	90
z	-154	-74	-40	-20	-6	2	-2	0

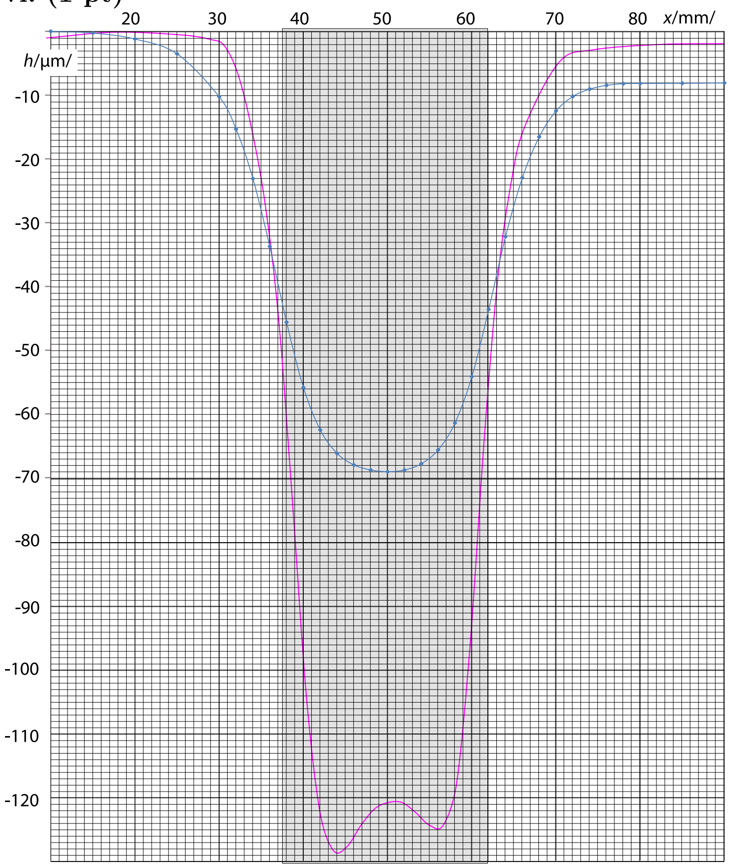
v. (1.6 pts) The water height can be obtained as the integral $h = \int \tan\beta dx$. Thus, we calculate the water height row-by-row, by adding to the height in the previous row the product of the horizontal displacement $x_{i+1} - x_i$ with the average slope $\frac{1}{2}(\tan\beta_{i+1} + \tan\beta_i)$.

x (mm)	10	15	20	25	30	32	34	36
$-h$ (μm)	0	0	1	4	10	15	23	34
x (mm)	38	40	42	44	46	48	50	52
$-h$ (μm)	46	56	63	66	68	69	69	69
x (mm)	54	56	58	60	62	64	66	68
$-h$ (μm)	68	66	61	54	44	32	23	17
x (mm)	70	72	74	76	78	80	85	90
$-h$ (μm)	12	10	9	8	8	8	8	8

Note that the water level height at the end of the table should be also 0 (this corresponds also to an unperturbed region); the non-zero result is explained by the measurement uncertainties. One can improve the result by subtracting from h a linear trend $8\,\mu\text{m} \cdot \frac{x-10\,\text{mm}}{80\,\text{mm}}$.

If the water level above the magnet is 1 mm, the water level descends below its unperturbed level at the axis of the magnet by ca $120\,\mu$m.

vi. (1 pt)



Similarly to the previous figure, blue curve corresponds to a water depth of *ca* 2 mm, (data given in the table above), and the violet one — to 1 mm.

The position of the magnet can be found by measuring the caliper (find the positions when the laser beam hits the edges of the magnet and determine the distance between these positions — the result is ca 24 mm), and using the symmetry: magnet is placed symmetrically with respect to the surface elevation curve.

Part C. Magnetic permeability *(2 points)*

Water surface takes an equipotential shape; for a unit volume of water, the potential energy associated with the magnetic interaction is $\frac{B^2}{2\mu_0}(\mu^{-1}-1) \approx B^2\frac{1-\mu}{2\mu_0}$; the potential energy associated with the Earth's gravity is $\rho g h$. At the water surface, the sum of those two needs to be constant; for a point at unperturbed surface, this expression equals to zero, so $B^2\frac{\mu-1}{2\mu_0} + \rho g h = 0$ and hence, $\mu - 1 = 2\mu_0\rho g h/B^2$. Here, $h = 120\,\mu$m stands for the depth of the water surface at the axis of the magnet; note that we have compensated the cumulative error as described at the end of the previous task and obtained h as the difference between the depth at the magnet's axis ($121\,\mu$m) and the half-depth at the right-hand-side of the graph ($1\,\mu$m). Putting in the numbers, we obtain $\mu - 1 = -1.2 \times 10^{-5}$.