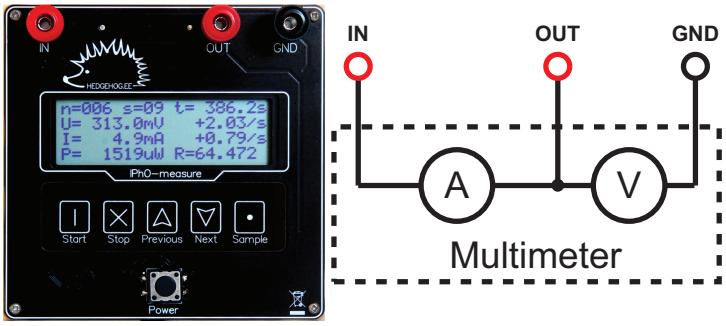
Problem E2. Nonlinear Black Box *(10 points)*

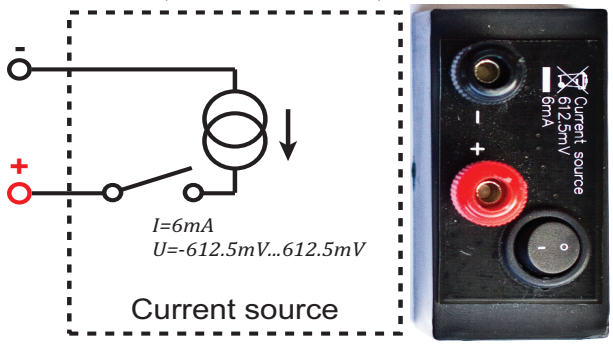
In simple problems, electrical circuits are assumed to consist of linear elements, for which electrical characteristics are directly proportional to each other. Examples include resistance ($V = RI$), capacitance ($Q = CV$) and inductance ($V = L\dot{I} = L\frac{dI}{dt}$), where R, C and L are constants. In this problem, however, we examine a circuit containing nonlinear elements, enclosed in a black box, for which the assumption of proportionality no longer holds.

The setup comprises a **multimeter** (labelled "IPhO-measure"), a black box that acts as a **current source**, a **black box** containing nonlinear elements, and four test leads with stackable connectors for wiring. Be careful not to break the seal on the black box.

The multimeter can measure current and voltage simultaneously. You can store with it up to 2000 data points, each consisting of: voltage V, current I, power $P = IV$, resistance $R = V/I$, voltage time derivative $\dot{V}$ ($= \frac{dV}{dt}$), current time derivative $\dot{I}$ ($= \frac{dI}{dt}$) and time t. See multimeter manual for details. If you go beyond 2000 stored data points, the oldest data will be overwritten.

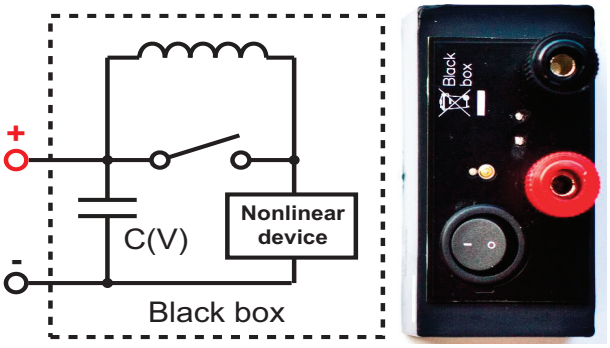


The constant current source supplies stable current as long as the voltage across its terminals stays between $-0.6125\,\mathrm{V}$ and $0.6125\,\mathrm{V}$. When switched off, the constant current source behaves as a large (essentially infinite) resistance.



The black box contains an electric double layer capacitor (which is a slightly nonlinear high capacitance capacitor), an unknown nonlinear element, and an inductor $L = 10\,\mu\mathrm{H}$ of negligible resistance, switchable as indicated on the circuit diagram. The nonlinear element can be considered as a resistance with a nonlinear dependence between the voltage and the current [$I(V)$ is a continuous function of V with $I(0) = 0$]. Likewise, for the capacitor, the differential capacitance $C(V) = dQ/dV$ is not exactly constant.

We say that the voltage on the black box is positive when the potential on its red terminal is higher than the potential on the black terminal. Positive voltage will be acquired when the terminals of matching colours on the black box and the current source are connected (you are allowed to use negative voltages).



It is safe to discharge the capacitor in the black box by shorting its inputs, either by itself or through the IN and OUT terminals on multimeter: the internal resistance of this capacitor is enough to keep the current from damaging anything.

You are not asked to estimate any uncertainties throughout this problem.

Part A. Circuit without inductance *(7 points)*

In this part, keep the switch on the black box closed (push "I" down), so that the inductance is shorted. **Please note** that some measurements may take a considerable time, therefore it is recommended that you read through all the tasks of part A to avoid unnecessary work.

i. (1.0 pts) Confirm that the output current of the current source is approximately 6 mA, and determine the range within which it varies for voltages between 0 and +480 mV. Document the circuit diagram used.

ii. (1.2 pts) Show that the differential capacitance $C(V)$ used in the black box is approximately 2 F by measuring its value for a single voltage of your choice $C(V_0) = C_0$. Document the circuit diagram.

iii. (2.2 pts) Neglecting the nonlinearity of the capacitance [$C(V) \approx C_0$], determine the current–voltage characteristic of the nonlinear element used in the black box. Plot the $I(V)$ curve for obtainable positive voltages on the black box onto the answer sheet. Document the circuit diagram.

iv. (2.6 pts) Using measurements taken from the whole range of obtainable voltages, calculate and plot the $C(V)$ curve for obtainable positive voltages from the black box on the answer sheet. Write down the minimal and maximal values of differential capacitance $C_{\min}, C_{\max}$. Document the circuit diagram.

Part B. Circuit with inductance *(3 points)*

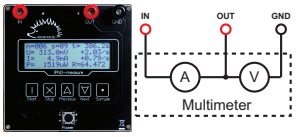
Enable the inductance by opening the switch on the black box (push "0" down). Using the same method as in pt. **A-iii**, measure and plot the current-voltage characteristic of the nonlinear element. Describe any significant differences between the curves of parts A and B and suggest a reason using qualitative arguments. You need to know that the nonlinear element also has a capacitance ($\approx 1\,\mathrm{nF}$) which is connected in parallel to the nonlinear resistance.

IPhO-measure: short manual

IPhO-measure is a multimeter capable of measuring voltage V and current I simultaneously. It also records their time derivatives $\dot{V}$ and $\dot{I}$, their product $P = VI$, ratio $R = V/I$, and time t of the sample. Stored measurements are organized into separate sets; every stored sample is numbered by the set number s and a counter n inside the set. All saved samples are written to an internal flash memory and can later be retrieved.

Electrical behaviour

The device behaves as an ammeter and a voltmeter connected as follows.



	Range	Internal resistance
Voltmeter	0 . . . 2 V	1 MΩ
Voltmeter	2 . . . 10 V	57 kΩ
Ammeter	0 . . . 1 A	1 Ω

Basic usage

- Push "Power" to switch the *IPhO-measure* on. The device is not yet measuring; to start measuring, push "start". Alternatively, you can now start browsing your stored data. See below.
- To browse previously saved samples (through all sets), press "Previous" or "Next". Hold them down longer to jump directly between sets.
- While not measuring, push "Start" to start measuring a new set.
- While measuring, push "Sample" each time you want to store a new set of data (i.e. of the readings shown on the display).
- While measuring, you can also browse other samples of the current set, using "Previous" and "Next".
- Press "Stop" to end a set and stop measuring. The device is still on. You are ready to start a new measuring session or start browsing stored data.
- Pushing "Power" turns the device off. The device will show text "my mind is going ..."; do not worry, all the data measurements will be stored and you will be able to browse them after you switch the device on again. Saved samples will not be erased.

Display

A displayed sample consists of nine variables:

1. index n of the sample in the set;
2. index s of the set;
3. time t since starting the set;
4. voltmeter output V;
5. rate of change of V (the time derivative $\dot{V}$); if derivative cannot be reliably taken due to fluctuations, "`+nan/s`" is shown;
6. ammeter output I;
7. rate of change of I (the time derivative $\dot{I}$); if derivative cannot be reliably taken due to fluctuations, "`+nan/s`" is shown;
8. product $P = VI$;
9. ratio $R = V/I$.

If any of the variables is out of its allowed range, its display shows "`+inf`" or "`-inf`".