1 teorinės eksperimento užduoties sprendimas

Parengė Gabija Žemaitytė

IPhO pasiruošimo stovykla 2015 06 25

Komentarai, papildantys sprendimą (oficialus sprendimas žemiau)

Pirma dalis:

1.1. Įtampos paklaidos sudedamos kvadratu.
Paliekamas vienas reikšminis skaitmuo.
Temperatūros paklaida - tiesiog padalos vertė.

1.2. Paklaidas turi: $V_{samp}(T_0)$, T_0, α, bet T neturi, nes jį pasirenkame.
Paklaidas randame išdiferencijavę:
$V_{samp}(T) = V_{samp}(T_0) - \alpha(T - T_0)$
Paliekame tik vieną skaitmenį kaip paklaidą:
$V_{samp}(50°C) = 524 \pm 4mV$

4.2. Kurioje vietoje žymėti kietėjimą grafike? Reikia naudotis visada (1) lygtimi iš sąlygos, o ne (3), nes žymiai lengviau nustatyti V_{samp}, o ne ΔV. Tuo labiau, kad kito abiejų diodų temperatūros, o turime tik vieną lygtį.

4.1. ir 4.2. Reikia parodyti, kad iš skirtingų grafikų gautos T_s.

4.3. Turi būti $T_s = 60 \pm 3°C$, o ne $T_s = 60 \pm 2.5°C$.
Antra dalis:

1. Temperatūros paklaida tik paskutinis skaitmuo po kablelio.

3.1. Jei turime

$$\Delta T(t) = \Delta T(0)e^{-\frac{k}{C}t}$$

Logaritmuojame:

$$\ln\left(\frac{\Delta T(t)}{\Delta T(0)}\right) = -\frac{k}{C}t$$

$$\ln(\Delta T(t)) - \ln(\Delta T(0)) = -\frac{k}{C}t$$

$\Delta T(t)$ randame iš (3) lygties uždavinyje:

$$\Delta V(t) = \Delta V(T_0) + \alpha\Delta T$$

$$\Delta T = (\Delta V(t) - \Delta V(T_0))/\alpha$$

Pakeitėm ženklus, nes mūsų diodai yra priešingi.
Įsistatant gauname:

$$\ln(\Delta V(t) - \Delta V(T_0)) - \ln(\alpha) - \ln(\Delta T(0)) = -\frac{k}{C}t$$

Pasirenkame paprasčiausius kintamuosius: $x = t$ ir $y = \ln(\Delta V(t) - \Delta V(T_0))$.

Pastaba: galima ir
$y = \ln(\frac{\Delta V(0) - \Delta V(T_0)}{\Delta V(t) - \Delta V(T_0)}) = \ln(\Delta V(0) - \Delta V(T_0)) - \ln(\Delta V(t) - \Delta V(T_0))$
kur $\ln(\Delta V(0) - \Delta V(T_0))$ - konstanta, bet tai tik pridėtų konstantą skaičiuojant, kas gali būti ne taip patogu skaičiuojant skaičiuotuvu.

3.2. Grafike reikia žymėti $y = \ln(\frac{\Delta V(t) - \Delta V(T_0)}{1mV})$, nes logaritmas turi būti bedimensinis.

Dažna klaida: $\Delta V(T_0)$ - kambario temperatūroje šiuo konkrečiu atveju. Nes pagal apibrėžimą, šis dydis yra dviejų diodų įtampų skirtumas esant toje pačioje temperatūroje. Šio eksperimento metu diodai toje pačioje temperatūroje buvo tik tada, kai buvo kambario temperatūros.

Grafikas nėra tiesinis dėl matavimų klaidos lentelėje arba dėl teorijos neatitikimo eksperimentui. Lentelėje buvo duotos įtampos, kurios yra mažesnės už kambario temperatūroje esantį įtampų skirtumą. Tai reikštų, kad lėkštelė atvėso žemiau kambario temperatūros, ko negali būti. Greičiausiai atraminis diodas buvo nepakankamai izoliuotas ir kažkiek įšilo. Arba eksperimentatorius surinko blogus duomenis. Tokiu atveju būtų sunku išsiapeliuoti, nes apkaltins jus, o ne įrangą.

3.3. Skaičiuojant iš grafiko Δy, nereikia daryti jokių pakeitimų į V ar mV, nes:
$\Delta y = ln(a) - ln(b) = ln(a/b)$ - dimensijos susiprastina.

Logaritmo paklaida $\delta(\ln(\Delta V(t) - \Delta V(T_0)))$.

4.1. Turime patikrinti formulę:

$$\Delta T(t) = \frac{\Phi}{k}\left(1 - e^{-\frac{k}{C}t}\right)$$

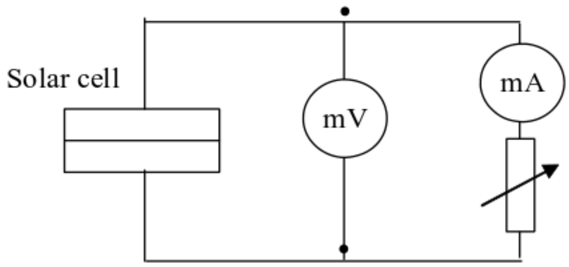
Kadangi prašo patikrinti formulę, mes negalime išskleisti Teiloro eilute eksponentės, nors k ir mažas.

Kintamuosius galima pasirinkti arba:
$x = 1 - e^{-\frac{k}{C}t}$, $y = \Delta V(t) - \Delta V(T_0)$,
arba:
x gali būti ir $x = e^{-\frac{k}{C}t}$.

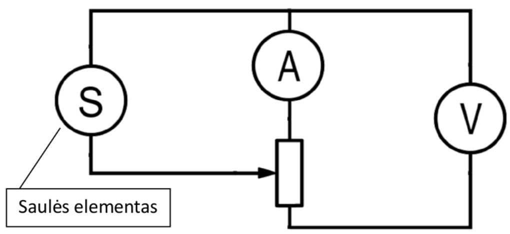
Galima uždėti modulį ant y, bet kam jis reikalingas?

Galima išlogaritmuot, kaip 3 klausime, ir gauti $x = t$, tik tas būdas nėra toks patogus, nes dar gauname papildomai logaritme $\frac{\Phi}{k}$.

5.1. Du būdai nubrėžti grandinę:



1 pav.: Grandinė, kai voltmetras tik ant saulės elemento.



2 pav.: Grandinė, kai voltmetras ant viso rezistoriaus.

Geriau brėžti pirmąjį variantą, nes praktiniu atveju voltmetro varža nėra begalinė, todėl tiksliau matuoti tiesiai ant saulės elemento.

Bendros pastabos:

Grafiko taškus galima sujungti tolydžia kreive, bet ne kampuotais laipteliais.

Solution

Task 1

1.

1.1. $T_0 =$ 25±1 °C

$$V_{\text{samp}}\left(T_0\right) = 573.9 \text{ mV}$$

With different experiment sets, V_{samp} may differ from the above value within ±40 mV.

Note for error estimation:
δV and δV are calculated using the specs of the multimeter: ±0.5% reading digit +2 on the last digit. Example: if V = 500mV, the error δV = 500×0.5% + 0.2 = 2.7 mV ≈ 3 mV.

Thus, $V_{samp}\left(T_0\right) = 574 \pm 3 \text{ mV}$.

All values of $V_{samp}\left(T_0\right)$ within 505÷585 mV are acceptable.

1.2. Formula for temperature calculation:

From Eq (1): $V_{\text{samp}} = V_{\text{samp}}(T_0) - \alpha(T - T_0)$

$$V_{\text{samp}}\left(50^{\text{o}}\text{C}\right) = 523.9 \text{ mV}$$

$$V_{\text{samp}}\left(70^{\text{o}}\text{C}\right) = 483.9 \text{ mV}$$

$$V_{\text{samp}}\left(80^{\text{o}}\text{C}\right) = 463.9 \text{ mV}$$

Error calculation: $\delta V_{\text{samp}} = \delta V_{\text{samp}}\left(T_0\right) + \left(T - T_0\right)\delta\alpha$

Example: $V_{\text{samp}} = 495.2 \text{ mV}$, then $\delta V_{\text{samp}} = 2.7 + 0.03 \times (50 - 25) = 3.45 \text{ mV} \approx 3.5 \text{ mV}$

Thus:

$$V_{\text{samp}}\left(50^{\text{o}}\text{C}\right) = 524 \pm 4 \text{ mV}$$

$$V_{\text{samp}}\left(70^{\text{o}}\text{C}\right) = 484 \pm 4 \text{ mV}$$

$$V_{samp}\left(80^{o}\mathrm{C}\right) = 464 \pm 5 \text{ mV}$$

The same rule for acceptable range of V_{samp} as in 1.1 is applied.

2.

2.1. Data of cooling-down process without sample:

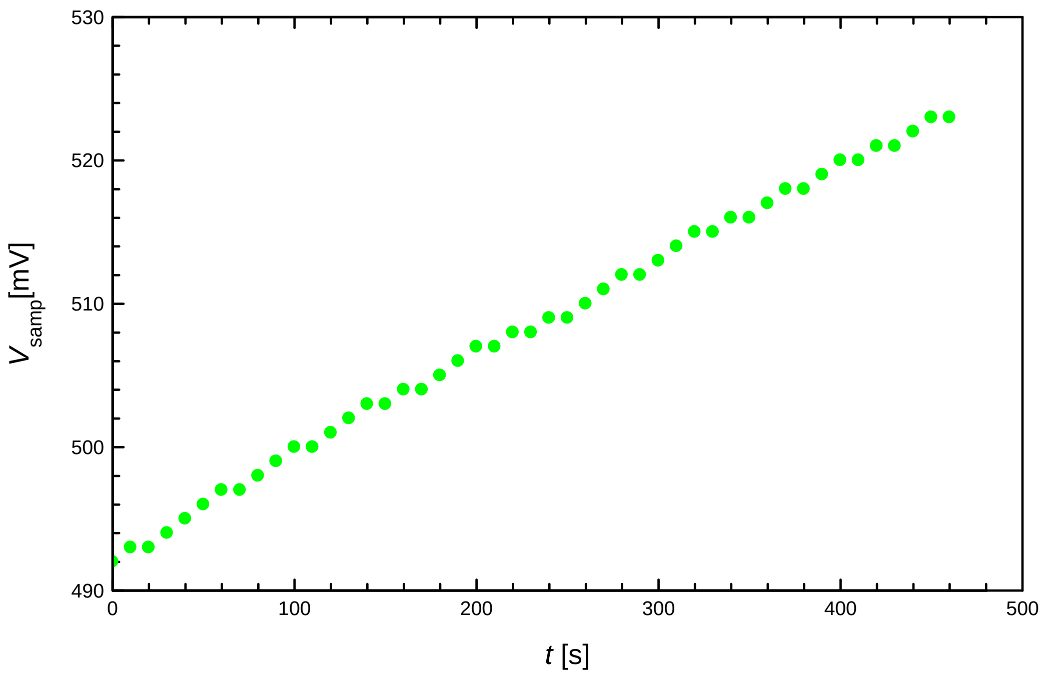
t (s)	V_{samp} (mV) (±3mV)	ΔV (mV) (±0.2mV)
0	492	-0.4
10	493	-0.5
20	493	-0.5
30	494	-0.6
40	495	-0.7
50	496	-0.7
60	497	-0.8
70	497	-0.8
80	498	-0.9
90	499	-1.0
100	500	-1.0
110	500	-1.1
120	501	-1.1
130	502	-1.2
140	503	-1.2
150	503	-1.3
160	504	-1.3
170	504	-1.4
180	505	-1.5
190	506	-1.6
200	507	-1.6
210	507	-1.7
220	508	-1.7
230	508	-1.8
240	509	-1.8
250	509	-1.8
260	510	-1.9
270	511	-1.9

280	512	-1.9
290	512	-2.0
300	513	-2.0
310	514	-2.1
320	515	-2.1
330	515	-2.1
340	516	-2.1
350	516	-2.2
360	517	-2.2
370	518	-2.3
380	518	-2.3
390	519	-2.3
400	520	-2.4
410	520	-2.4
420	521	-2.5
430	521	-2.5
440	522	-2.5
450	523	-2.6
460	523	-2.6

The acceptable range of ΔV is ±40 mV. There is no fixed rule for the change in ΔV with T (this depends on the positions of the dishes on the plate, etc.)

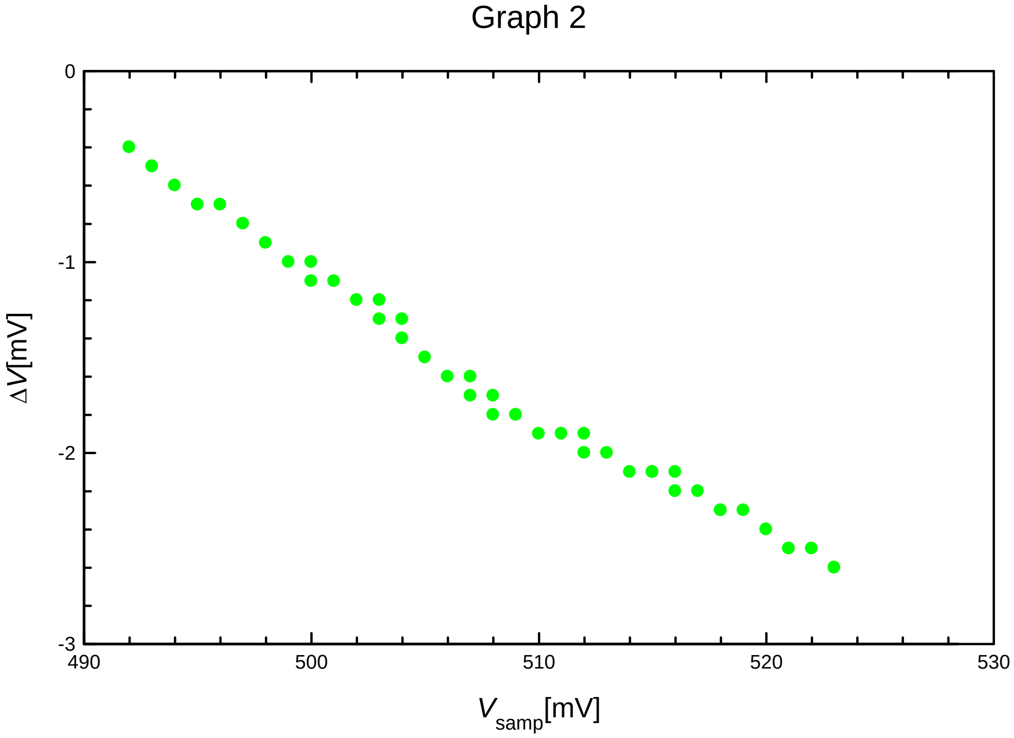
2.2.

Graph 1



The correct graph should not have any abrupt changes of the slope.

2.3.



The correct graph should not have any abrupt changes of the slope.

3.

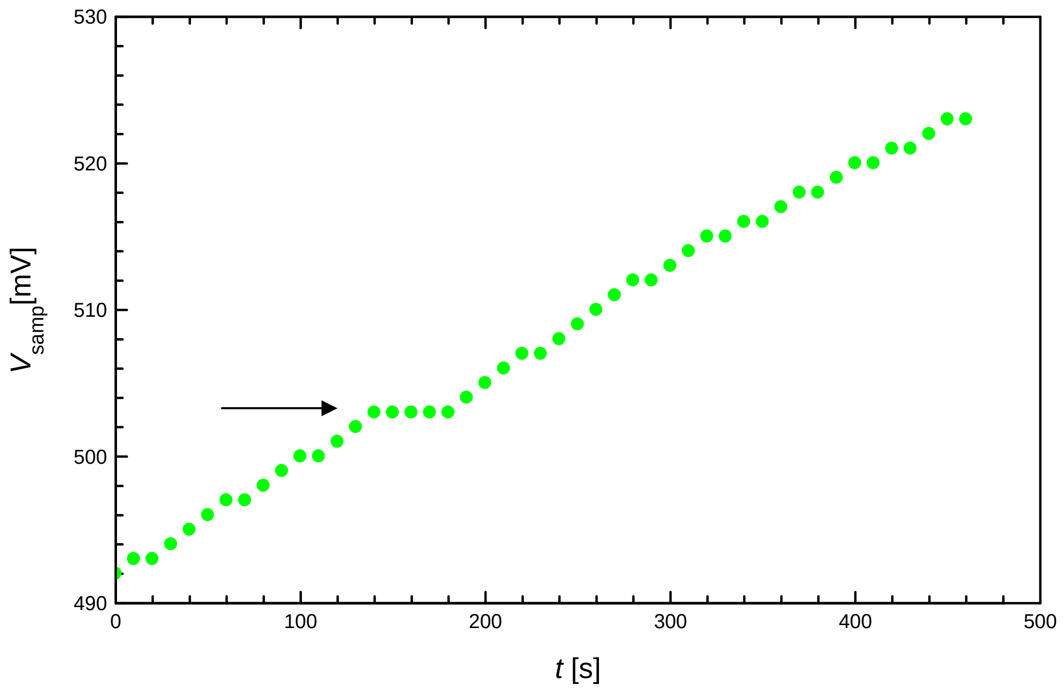
3.1. Dish with substance

t (s)	V_{samp} (mV) (±3mV)	ΔV (mV) (±0.2mV)
0	492	-4.6
10	493	-4.6
20	493	-4.6
30	494	-4.6
40	495	-4.6
50	496	-4.6
60	497	-4.6
70	497	-4.5
80	498	-4.5
90	499	-4.5
100	500	-4.5
110	500	-4.5
120	501	-4.5

130	502	-4.6
140	503	-4.6
150	503	-5.1
160	503	-5.6
170	503	-6.2
180	503	-6.5
190	504	-6.6
200	505	-6.5
210	506	-6.4
220	507	-6.3
230	507	-6.1
240	508	-5.9
250	509	-5.7
260	510	-5.5
270	511	-5.3
280	512	-5.1
290	512	-5.0
300	513	-4.9
310	514	-4.8
320	515	-4.7
330	515	-4.7
340	516	-4.6
350	516	-4.6
360	517	-4.5
370	518	-4.5
380	518	-4.4
390	519	-4.4
400	520	-4.4
410	520	-4.4
420	521	-4.4
430	521	-4.3
440	522	-4.3
450	523	-4.3
460	523	-4.3

3.2.

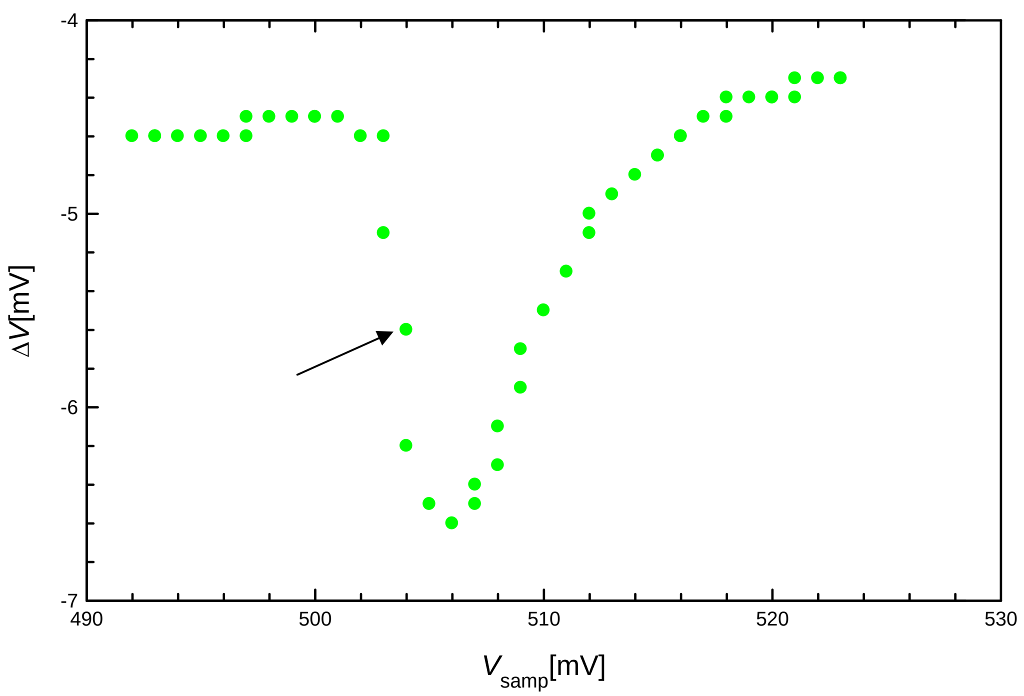
Graph 3



The correct Graph 3 should contain a short plateau as marked by the arrow in the above figure.

3.3.

Graph 4



The correct Graph 4 should have an abrupt change in ΔV, as shown by the arrow in the above figure.

Note: when the dish contains the substance, values of ΔV may change compared to those without the substance.

4.

4.1. V_s is shown in Graph 3. Value V_s = (503±3) mV. From that, T_s = 60.5 ^{o}C can be deduced.

4.2. V_s is shown in Graph 4. Value V_s = (503±3) mV. From that, T_s = 60.5 ^{o}C can be deduced.

4.3. Error calculations, using root mean square method:

Error of T_s: $T_s = T_0 + \frac{V(T_0) - V(T_s)}{\alpha} = T_0 + A$, in which A is an intermediate variable.

Therefore error of T_s can be written as $\delta T_s = \sqrt{(\delta T_0)^2 + (\delta A)^2}$, in which δ... is the error.

Error for A is calculated separately:

$$\delta A = \frac{V(T_0) - V(T_s)}{\alpha}\sqrt{\left\{\frac{\delta[V(T_0) - V(T_s)]}{V(T_0) - V(T_s)}\right\}^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

in which we have:

$$\delta[V(T_0) - V(T_s)] = \sqrt{[\delta V(T_0)]^2 + [\delta V(T_s)]^2}$$

Errors of other variables in this experiment:

$\delta T_0 = 1^oC$

$\delta V(T_0)$ = 3 mV, read on the multimeter.

$\delta\alpha$ = 0.03 mV/^{o}C

$\delta V(T_s) \approx$ 3 mV

From the above constituent errors we have:

$$\delta[V(T_0) - V(T_s)] \approx 4.24\,mV$$

$\delta A \approx 2.1°C$

Finally, the error of T_s is: $\delta T_s \approx 2.5°C$

Hence, the final result is: T_s =60±2.5 ^oC

Note: if the student uses any other reasonable error calculation method that leads to approximately the same result, it is also accepted.

Task 2

1.

1.1. $T_0 = 26 \pm 1^{o}C$

2.

2.1. Measured data with the lamp off

t (s)	$\Delta V(T_0)$ (mV) (±0.2mV)
0	19.0
10	19.0
20	19.0
30	19.0
40	19.0
50	18.9
60	18.9
70	18.9
80	18.9
90	18.9
100	19.0
110	19.0
120	19.0

Values of $\Delta V(T_0)$ can be different from one experiment set to another. The acceptable values lie in between -40÷+40 mV.

2.2. Measured data with the lamp on

t (s)	ΔV (mV) (±0.2mV)
0	19.5
10	21.9
20	23.8
30	25.5
40	26.9
50	28.0
60	29.0
70	29.9
80	30.7
90	31.4

100	32.0
110	32.4
120	32.9

When illuminated (by the lamp) values of ΔV may change 10 ÷ 20 mV compared to the initial situation (lamp off).

2.3. Measured data after turning the lamp off

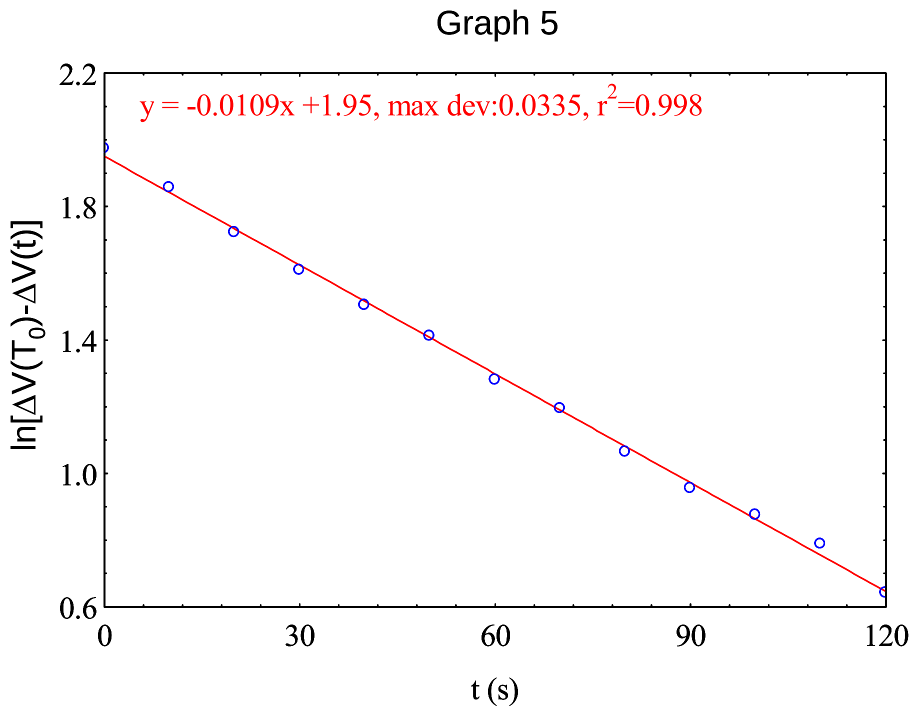
t (s)	ΔV (mV) (±0.2mV)
0	23.2
10	22.4
20	21.6
30	21.0
40	20.5
50	20.1
60	19.6
70	19.3
80	18.9
90	18.6
100	18.4
110	18.2
120	17.9

3. Plotting graph 5 and calculating k

3.1. $x = t\,;\ \ y = \mathbf{ln}\left[\Delta V\left(T_0\right) - \Delta V\left(t\right)\right]$

Note: other reasonable ways of writing expressions for x and y that also leads to a linear relationship using **ln** are also accepted.

3.2. Graph 5



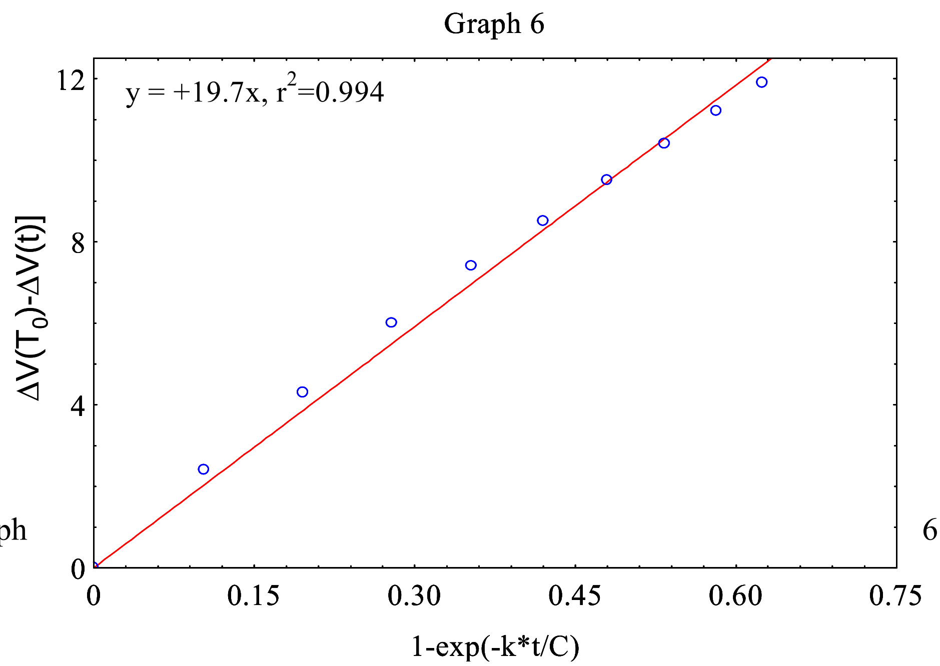
3.3. Calculating k: $\frac{k}{C}=$ 0.0109 s^{-1} and $C = 0.69$ J/K, thus: $k = 7.52\times10^{-3}$ W/K

Note: Error of k will be calculated in 5.5. Students are not asked to give error of k in this step. The acceptable value of k lies in between $6\times10^{-3} \div 9\times10^{-3}$ W/K depending on the experiment set.

4. Plotting Graph 6 and calculating E

4.1. $x=\left[1-\exp\left(\frac{-kt}{C}\right)\right]$; $y=\left|\Delta V\left(T_0\right)-\Delta V\left(t\right)\right|$

4.2.



Graph 6 should

be substantially linear, with the slope in between 15÷25 mV, depending on the experiment set.

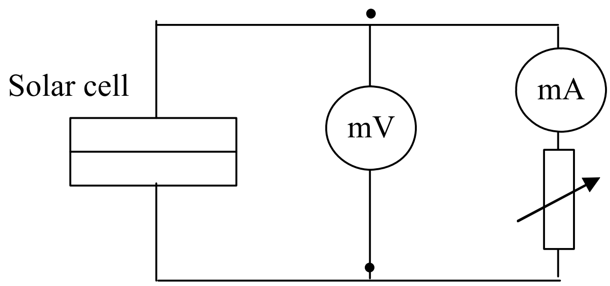
4.3. From the slope of Graph 6 and the area of the detector orifice we obtain E = 140 W/m^2. The area of the detector orifice is

$$S_{\text{det}} = \pi R_{\text{det}}^{\;2} = \pi \times (13\times 10^{-3})^2 = 5.30\times 10^{-4}\ \text{m}^2 \quad \text{with error: } \frac{\delta R_{\text{det}}}{R_{\text{det}}} = 5\%$$

Error of E will be calculated in 5.5. Students are not asked to give error of E in this step. The acceptable value of E lies in between 120 ÷ 160 W/m^2, depending on the experiment set.

5.

5.1. Circuit diagram:



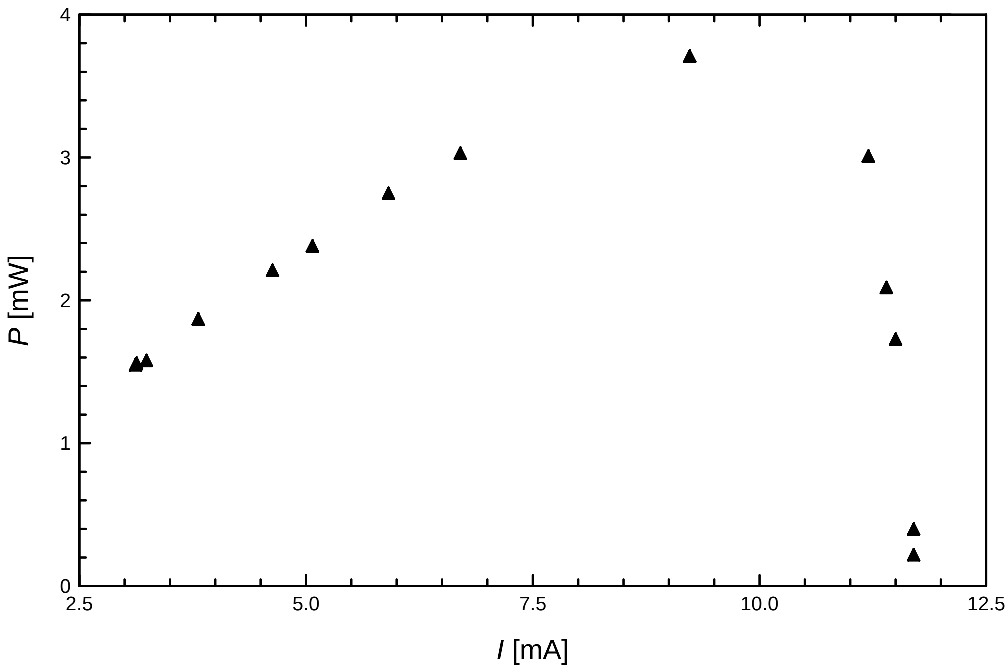
5.2. Measurements of V and I

V (mV) (±0.3÷3mV)	I (mA) (±0.05÷0.1mA)	P (mW)
18.6 ±0.3	11.7	0.21
33.5	11.7	0.39
150	11.5	1.72
157	11.6	1.82
182 ±1	11.4	2.08
267	11.2	3.00
402 ±2	9.23	3.70
448	6.70	3.02
459	5.91	2.74
468	5.07	2.37
473 ±3	4.63	2.20
480	3.81	1.86
485	3.24	1.57

487	3.12	1.54
489	3.13	1.55

5.3.

Graph 7



5.4. $P_{max} = 3.7 \pm 0.2$ mW

The acceptable value of P_{max} lies in between 3÷4.5 mW, depending on the experiment set.

5.5. Expression for the efficiency

$$S_{cell} = 19 \times 24\,\text{mm}^2 = 450 \times 10^{-6}\,\text{m}^2$$

Then $\eta_{max} = \dfrac{P_{max}}{E \times S_{cell}} = 0.058$

Error calculation:

$$\delta\eta_{max} = \eta_{max}\sqrt{\left(\frac{\delta P_{max}}{P_{max}}\right)^2 + \left(\frac{\delta E}{E}\right)^2 + \left(\frac{\delta S_{cell}}{S_{cell}}\right)^2}$$, in which S_{cell} is the area of the solar cell.

$\dfrac{\delta P_{max}}{P_{max}}$ is estimated from Graph 7, typical value ≈ 6 %

$\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}$: error from the millimeter measurement (with the ruler), typical value ≈ 5 %

E is calculated from averaging the ratio (using Graph 6):

$$B = \frac{\Delta V(T_0) - \Delta V(t)}{1 - \exp\left(-\frac{k}{C}t\right)} = \frac{E\pi R_{\text{det}}{}^2\alpha}{k}$$

in which B is an intermediate variable, R_{det} is the radius of the detector orifice.

$$E = \frac{kB}{\pi R_{\text{det}}{}^2\alpha}$$

Calculation of error of E:

$$\overline{\left(\frac{\delta E}{E}\right)} = \sqrt{\left(\frac{\delta k}{k}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\text{det}}}{R_{\text{det}}}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

k is calculated from the regression of:

$$\Delta T = \Delta T(0)\exp\left(-\frac{k}{C}t\right), \text{ hence } \ln\Delta T = \ln\Delta T(0) - \frac{k}{C}t$$

We set $k/C = m$ then $k = mC$

From the regression, we can calculate the error of m:

$$\frac{\delta m}{m} \approx 2(1-r) \approx 0.2\%$$

$$\frac{\delta k}{k} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2}$$

We derive the expression for the error of $\eta_{\max}$:

$$\delta\eta_{\max} = \eta_{\max}\sqrt{\left(\frac{\delta P_{\max}}{P_{\max}}\right)^2 + \left(\frac{\delta S_{\text{cell}}}{S_{\text{cell}}}\right)^2 + \left(\frac{\delta B}{B}\right)^2 + 4\left(\frac{\delta R_{\text{det}}}{R_{\text{det}}}\right)^2 + \left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\frac{\delta\alpha}{\alpha}\right)^2}$$

Typical values for $\eta_{\max}$ and other constituent errors:

$$\eta_{\max} \approx 0.058$$

$$\frac{\delta P_{\max}}{P_{\max}} = 5\% \ ; \quad \frac{\delta B}{B} \approx 0.6\% \ ; \quad \frac{\delta m}{m} \approx 0.2\% \ ; \quad \frac{\delta S_{\text{cell}}}{S_{\text{cell}}} \approx 5\% \ ; \quad \frac{\delta R_{\text{det}}}{R_{\text{det}}} \approx 5\% \ ;$$

$$\frac{\delta C}{C} \approx 3\% \,; \frac{\delta k}{k} \approx 3\% \,; \ \frac{\delta E}{E} \approx 10.5\% \,; \ \frac{\delta \alpha}{\alpha} \approx 1.5\%$$

Finally:

$$\frac{\delta \eta_{\max}}{\eta_{\max}} = 12.7\% \,; \ \ \delta\eta_{\mathbf{max}} \approx 0.0074$$

and

$$\eta_{\mathbf{max}} = (5.8 \pm 0.8)\%$$

Note: if the student uses any other reasonable error method that leads to approximately the same result, it is also accepted.