2 teorinės eksperimento užduoties sprendimas

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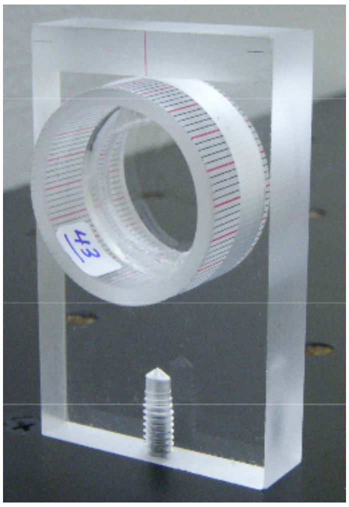
IPhO pasiruošimo stovykla 2015 06 25

Komentarai, papildantys sprendimą (oficialus sprendimas žemiau)

1. Tikslaus nulio nustatymas.

Problema: matuojame šviesos intensyvumo priklausomybę nuo kampo apskrita liniuote (Pav. 1). Gauname kažką panašaus į parabolę. Tačiau intensyvumo minimumas nėra tiksliai ties liniuotės padala. Jei rasime korekciją, galėsime tiksliau perskaičiuoti kitus duomenis.



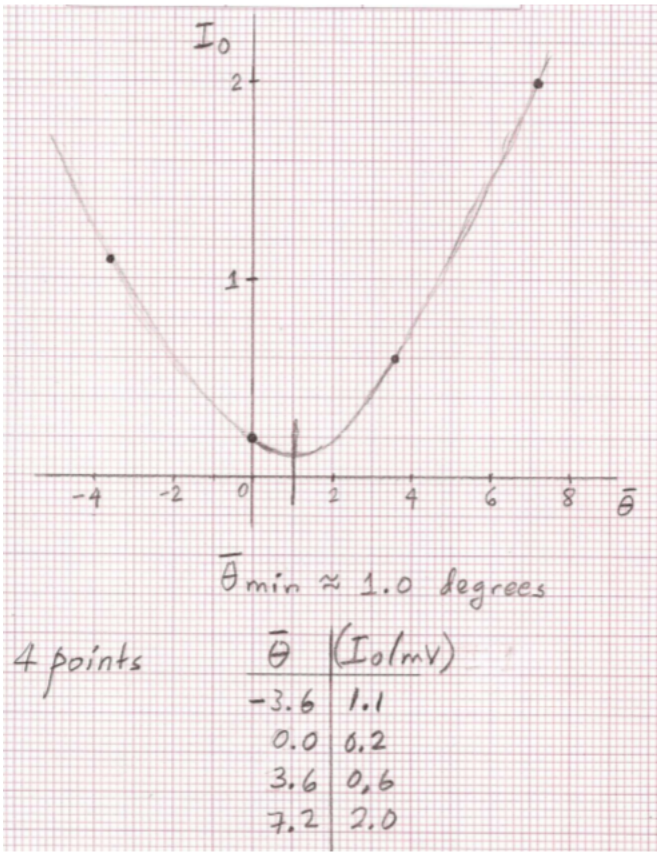
1 pav.: Apvali liniuotė.

Tai galima padaryti dviem būdais.

I būdas: grafinis. Nusibrėžiam intensyvumo grafiką, kuris yra parabolė, per 4 ar truputį daugiau taškų ir iš akies nustatome, kur yra jos minimumas (Pav. 2). Pasirinkti taškai turi būti netoli minimumo.

II būdas: grafiko minimumą galime tarti esant parabolę, todėl pasiimame tris taškus (x, y) ir iš lygties: $y = ax^2 + bx + c$ išskaičiuojame koeficientus a, b, c. Pasiimti taškai turi būti netoli minimumo. $x_{min} = -\frac{b}{2a}$.

2. Kintamųjų keitimas.



2 pav.: Liniuotės nulinės padalos nustatymas iš grafiko.

Turime lygtį $I_0(\theta) = 0.5(1 - \cos(\Delta\phi)) \sin^2(2\theta)$, kur $\Delta\phi$ konstanta.
Pasirenkame $y = I_0(\theta)$, o $x = \sin^2(2\theta)$.

3. Reikia atidumo, kai kampas, kuris gali būti matuojamas 2π rad periodu, yra lygus kitų fizikinių dydžių funkcijai. Pvz.:

$$\Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2|$$

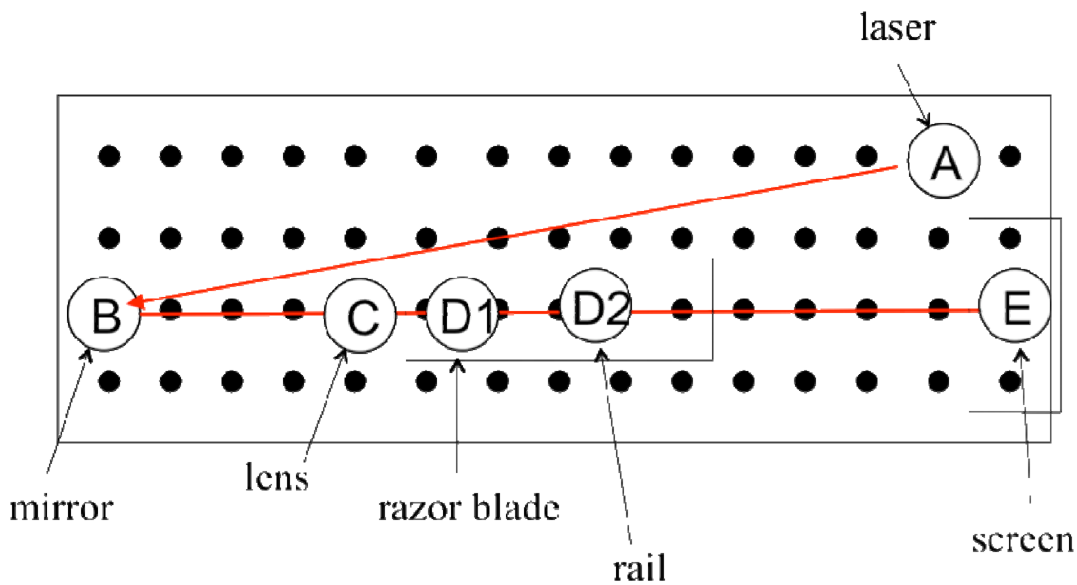
Čia $\Delta\phi$ - fazių skirtumas tarp lygiagrečiai ir statmenai poliarizuotų bangų.
Kairė lygties pusė gali būti $2\pi - \Delta\phi$, priklausomai nuo L.

Answer Form

Experimental Problem No. 1

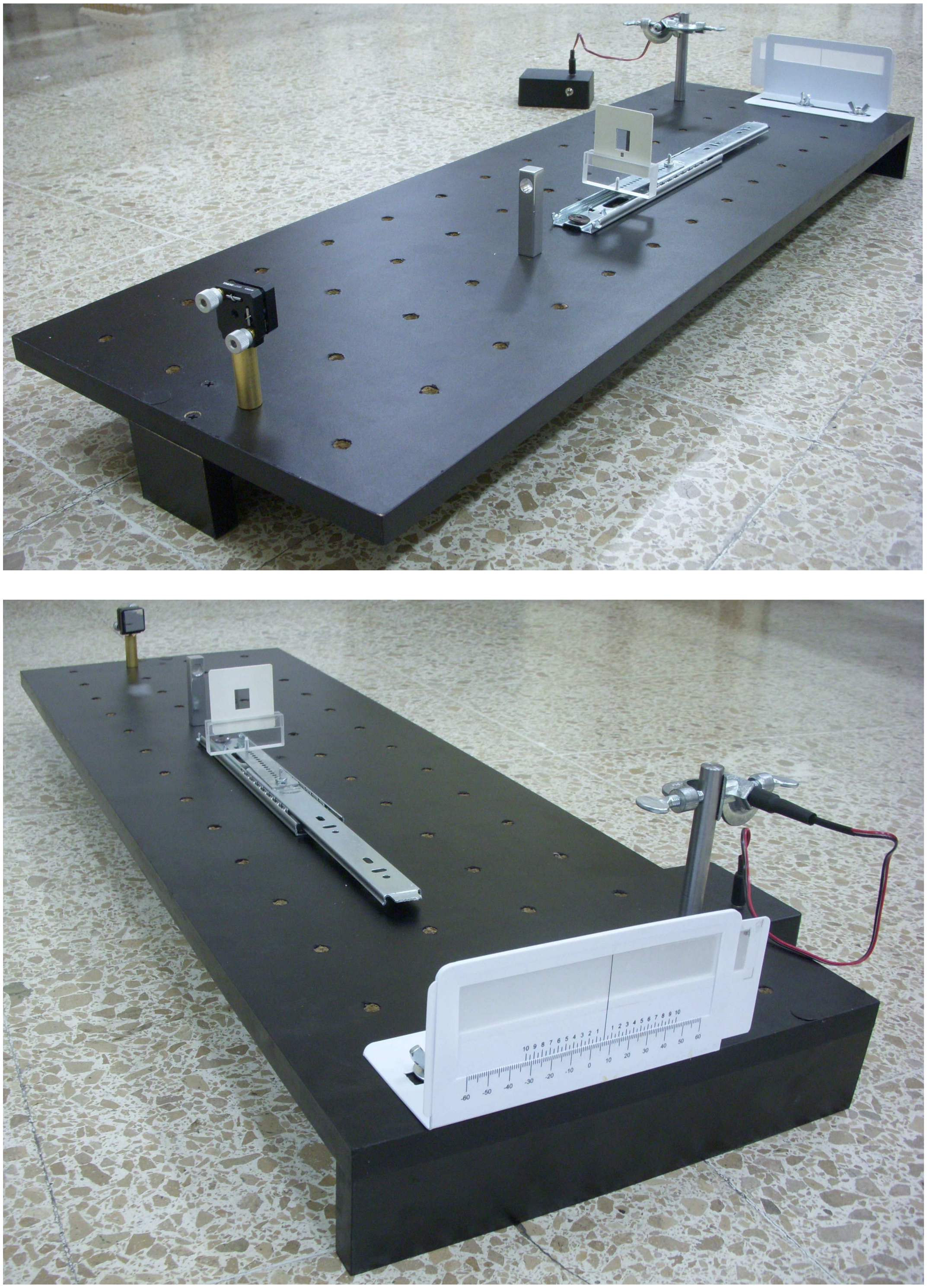
Diode laser wavelength

Task 1.1 Experimental setup.



(0.75)

1.1	Sketch the laser path in drawing of Task 1.1 and Write down the height h of the beam as measured from the table $h \pm \Delta h = (5.0 \pm 0.05) \times 10^{-2}$ m (0.25)	1.0



Experimental setup for measurement of diode laser wavelength

Task 1.2 Expressions for optical path differences.

1.2	The path differences are Case I: (0.25) $\Delta_{\mathrm{I}}(n) = (BF + FP) - BP = (L_b - L_0) + \sqrt{L_0^2 + L_R^2(n)} - \sqrt{L_b^2 + L_R^2(n)}$ $= (L_b - L_0) + L_0\sqrt{1 + \frac{L_R^2(n)}{L_0^2}} - L_b\sqrt{1 + \frac{L_R^2(n)}{L_b^2}}$ using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ $\approx (L_b - L_0) + L_0\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_0^2}\right) - L_b\left(1 + \frac{1}{2}\frac{L_R^2(n)}{L_b^2}\right)$ $\Rightarrow \quad \Delta_{\mathrm{I}}(n) \approx \frac{1}{2}L_R^2(n)\left(\frac{1}{L_0} - \frac{1}{L_b}\right)$ Case II: (0.25) $\Delta_{\mathrm{II}}(n) = (FB + BP) - FP = (L_0 - L_a) + \sqrt{L_a^2 + L_L^2(n)} - \sqrt{L_0^2 + L_L^2(n)}$ $\approx (L_0 - L_a) + L_a\sqrt{1 + \frac{L_L^2(n)}{L_a^2}} - L_0\sqrt{1 + \frac{L_L^2(n)}{L_0^2}}$ using $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ $\approx (L_0 - L_a) + L_a\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_a^2}\right) - L_0\left(1 + \frac{1}{2}\frac{L_L^2(n)}{L_0^2}\right)$ $\Rightarrow \quad \Delta_{\mathrm{II}}(n) \approx \frac{1}{2}L_L^2(n)\left(\frac{1}{L_a} - \frac{1}{L_0}\right)$	0.5

Task 1.3 Measuring the dark fringe positions and locations of the blade. Use additional sheets if necessary.

TABLE I

n	$(l_R(n) \pm 0.1) \times 10^{-3}$ m	$(l_L(n) \pm 0.1) \times 10^{-3}$ m	x_R	x_L
0	-7.5	1.1	0.791	0.935
1	-10.1	3.7	1.275	1.369
2	-12.4	6.4	1.620	1.696
3	-14.0	8.2	1.903	1.968
4	-15.6	10.0	2.151	2.208
5	-17.2	11.4	2.372	2.424
6	-18.4	12.2	2.574	2.622
7	-19.7		2.761	
8	-20.7		2.937	
9	-22.0		3.102	
10	-23.0		3.260	
11	-24.1		3.410	

1.3	Report positions of the blade and their difference with higher precision: $L_b \pm \Delta L_b = (653 \pm 1)\times 10^{-3}$ m (0.25) LABEL (I) (measuring tape) $L_a \pm \Delta L_a = (628 \pm 1)\times 10^{-3}$ m (0.25) LABEL (I) (measuring tape) $d = L_b - L_a = (24.6 \pm 0.1)\times 10^{-3}$ m (0.25) LABEL (H) (caliper)	3.25

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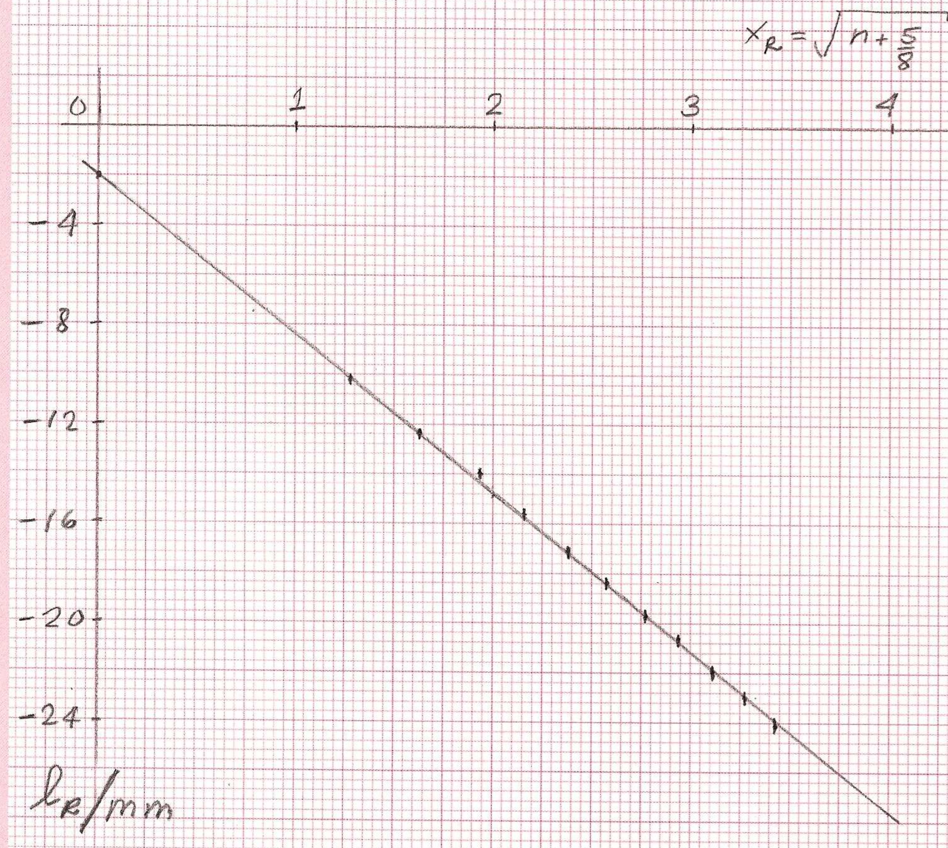
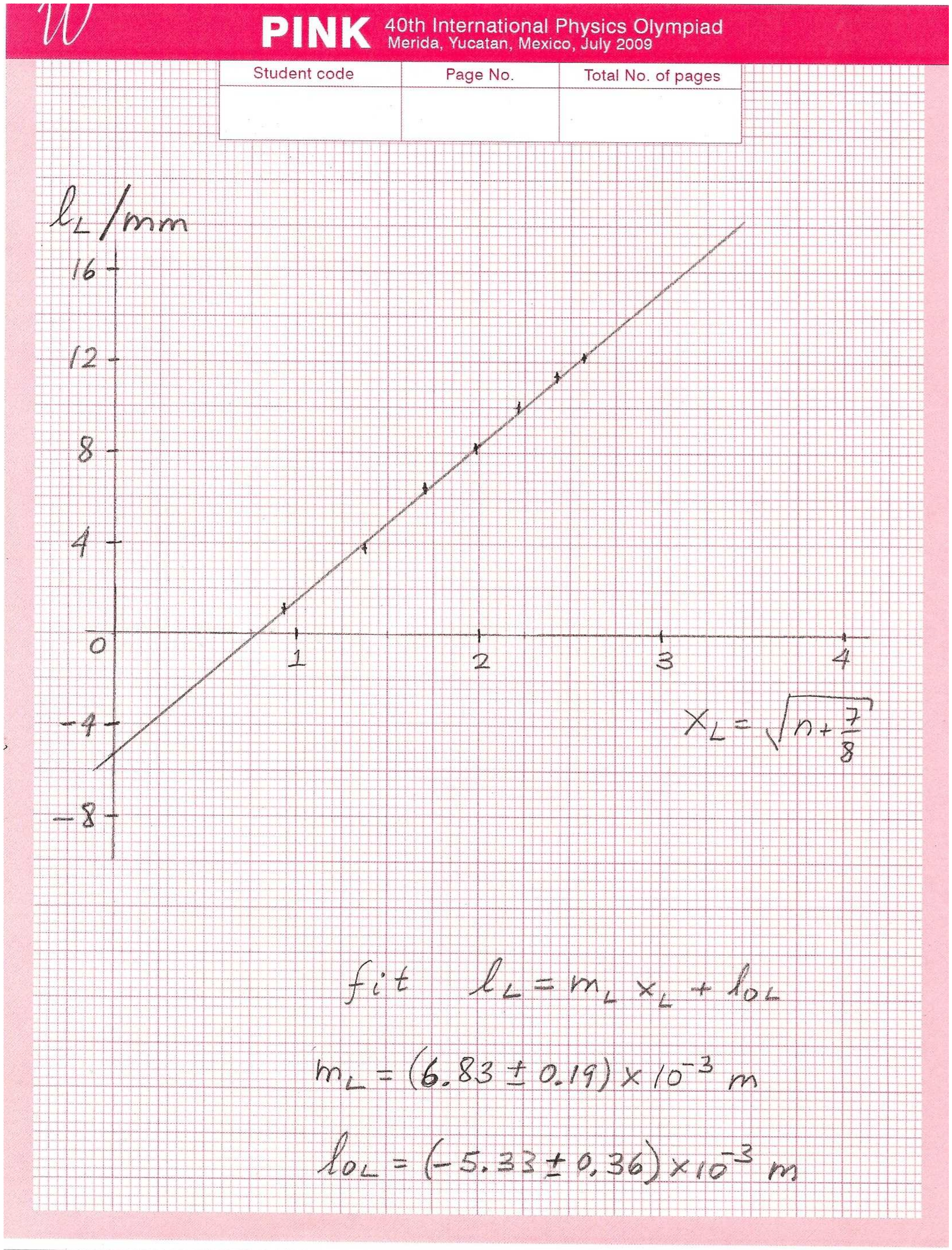


fit $l_R = m_R x_R + l_{0R}$

$m_R = (-6.39 \pm 0.07) \times 10^{-3}$ m

$l_{0R} = (-2.06 \pm 0.17) \times 10^{-3}$ m



Task 1.4 Performing a statistical and graphical analysis.

<table>
<tr><td>1.4</td><td>
A procedure:

From the condition of dark fringes and Task 1.2, we have

$$\frac{1}{2}L_R^2(n)\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$$

and

$$\frac{1}{2}L_L^2(n)\left(\frac{1}{L_a}-\frac{1}{L_0}\right)=\left(n+\frac{7}{8}\right)\lambda$$

Using (1.5), $L_R(n)=l_R(n)-l_{0R}$ and $L_L(n)=l_L(n)-l_{0L}$ we can rewrite

$$\frac{1}{2}(l_R(n)-l_{0R})^2\left(\frac{1}{L_0}-\frac{1}{L_b}\right)=\left(n+\frac{5}{8}\right)\lambda$$
$$\Rightarrow \quad l_R(n)=\sqrt{\frac{2L_bL_0}{L_b-L_0}\lambda}\sqrt{n+\frac{5}{8}}+l_{0R}$$

and

$$\frac{1}{2}(l_L(n)-l_{0L})^2\left(\frac{1}{L_a}-\frac{1}{L_0}\right)=\left(n+\frac{7}{8}\right)\lambda$$
$$\Rightarrow \quad l_L(n)=\sqrt{\frac{2L_aL_0}{L_0-L_a}\lambda}\sqrt{n+\frac{7}{8}}+l_{0L}$$

These can be cast as equations of a straight line, $y=mx+b$.

Case I:

$y_R=l_R \qquad x_R=\sqrt{n+\frac{5}{8}} \qquad m_R=\sqrt{\frac{2L_bL_0}{L_b-L_0}\lambda} \qquad b_R=l_{0R}$

Case II:

$y_L=l_L \qquad x_L=\sqrt{n+\frac{7}{8}} \qquad m_L=\sqrt{\frac{2L_aL_0}{L_0-L_a}\lambda} \qquad b_L=l_{0L}$

Perform least squares analysis of above equations. In Table I, we write down the values x_R and x_L.

One finds:

$m_R\pm\Delta m_R=(-6.39\pm0.07)\times10^{-3}$ m
</td><td>3.25</td></tr>
</table>

	$m_L \pm \Delta m_L = (6.83 \pm 0.19) \times 10^{-3}$ m and (values of l_{0R} and l_{0L}) $l_{0R} \pm \Delta l_{0R} = b_R \pm \Delta b_R = (-2.06 \pm 0.17) \times 10^{-3}$ m $l_{0L} \pm \Delta l_{0L} = b_L \pm \Delta b_L = (-5.33 \pm 0.36) \times 10^{-3}$ m The equations used in the least squares analysis: $m = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} y_{n'}}{\Delta}$ $b = \frac{\sum_{n=1}^{N} x_n^2 \sum_{n'=1}^{N} y_{n'} - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} x_{n'} y_{n'}}{\Delta}$ where $\Delta = N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2$ with N the number of data points. The uncertainty is calculated as $(\Delta m)^2 = N\frac{\sigma^2}{\Delta}$, $(\Delta b)^2 = \frac{\sigma^2}{\Delta}\sum_{n=1}^{N} x_n^2$ with, $\sigma^2 = \frac{1}{N-2}\sum_{n=1}^{N} (y_n - b - m x_n)^2$ REFERENCE: P.R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, 1969.	

Task 1.5 Calculating λ.

1.5	From any slope and the value of L_0 one finds, $\lambda = \frac{L_b - L_a}{2L_a L_b} \frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$ Using the suggestion to replace $d = L_b - L_a$, we can write	2.0

$$\lambda = \frac{d}{2L_a L_b} \frac{m_R^2 m_L^2}{m_R^2 + m_L^2}$$

$$\lambda \pm \Delta\lambda = (663 \pm 25) \times 10^{-9} \text{ m}$$

The uncertainty may range from 15 to 30 nanometers.

A precise measurement of the wavelength is $\lambda \pm \Delta\lambda = (655 \pm 1) \times 10^{-9}$ m.

The formula for the uncertainty,

$$\Delta\lambda = \sqrt{\left(\frac{\partial\lambda}{\partial d}\right)^2 \Delta d^2 + \left(\frac{\partial\lambda}{\partial L_a}\right)^2 \Delta L_a^2 + \left(\frac{\partial\lambda}{\partial L_b}\right)^2 \Delta L_b^2 + \left(\frac{\partial\lambda}{\partial m_R}\right)^2 \Delta m_R^2 + \left(\frac{\partial\lambda}{\partial m_L}\right)^2 \Delta m_L^2}$$

one finds,

$$\frac{\partial\lambda}{\partial d} = \frac{\lambda}{d}, \quad \frac{\partial\lambda}{\partial L_b} = \frac{\lambda}{L_b}, \quad \frac{\partial\lambda}{\partial L_a} = \frac{\lambda}{L_a} \quad \text{and} \quad \frac{\partial\lambda}{\partial m_R} = \frac{2m_L^2}{m_R} \frac{\lambda}{m_L^2 + m_R^2}$$

and analogously for the other slope.

One can calculate directly these quantities. However, one may note that the errors due to L_a, L_b and d are negligible. Moreover, $m_R^2 \approx m_L^2$ and $L_a \approx L_b$. This implies,

$$\frac{\partial\lambda}{\partial m_R} \approx \frac{\lambda}{m_R} \approx \frac{\partial\lambda}{\partial m_L}$$. Thus,

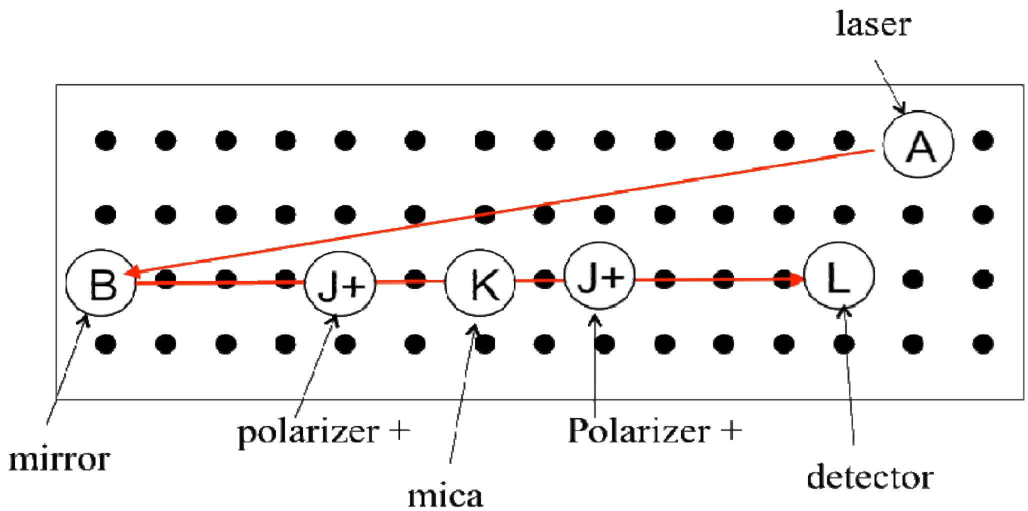
$$\Delta\lambda \approx \sqrt{2}\frac{\lambda}{m_L}\Delta m_L \approx \left(25 \times 10^{-9}\right) \text{ m}$$

Answer Form

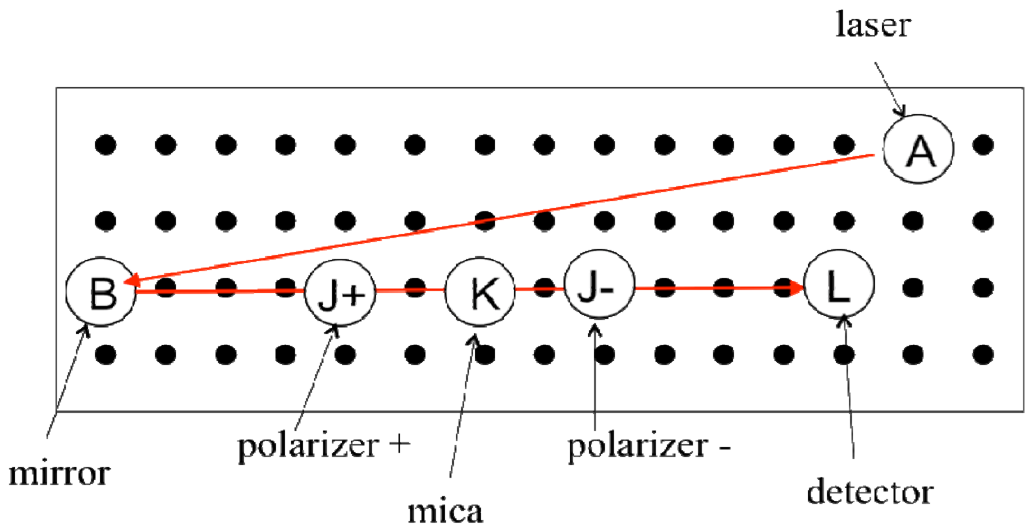
Experimental Problem No. 2

Birefringence of mica

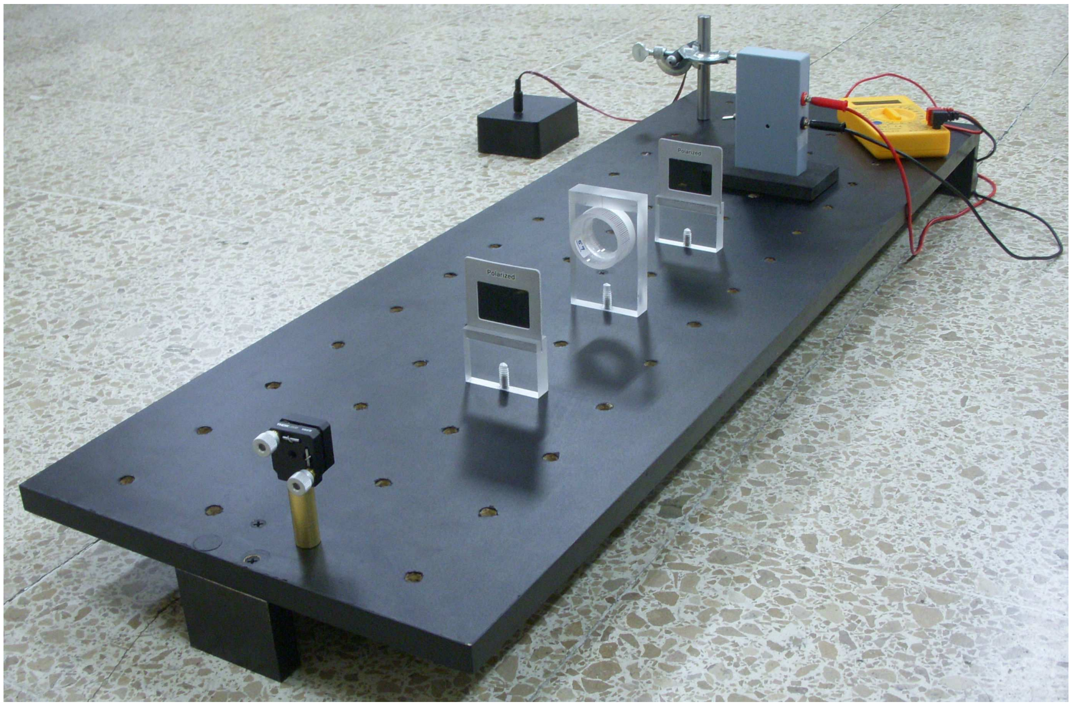
Task 2.1 a) Experimental setup for I_P. (0.5 points)



Task 2.1 b) Experimental setup for I_O. (0.5 points)



2.1		1.0



Experimental setup for measurement of mica birefringence

Task 2.2 The scale for angles.

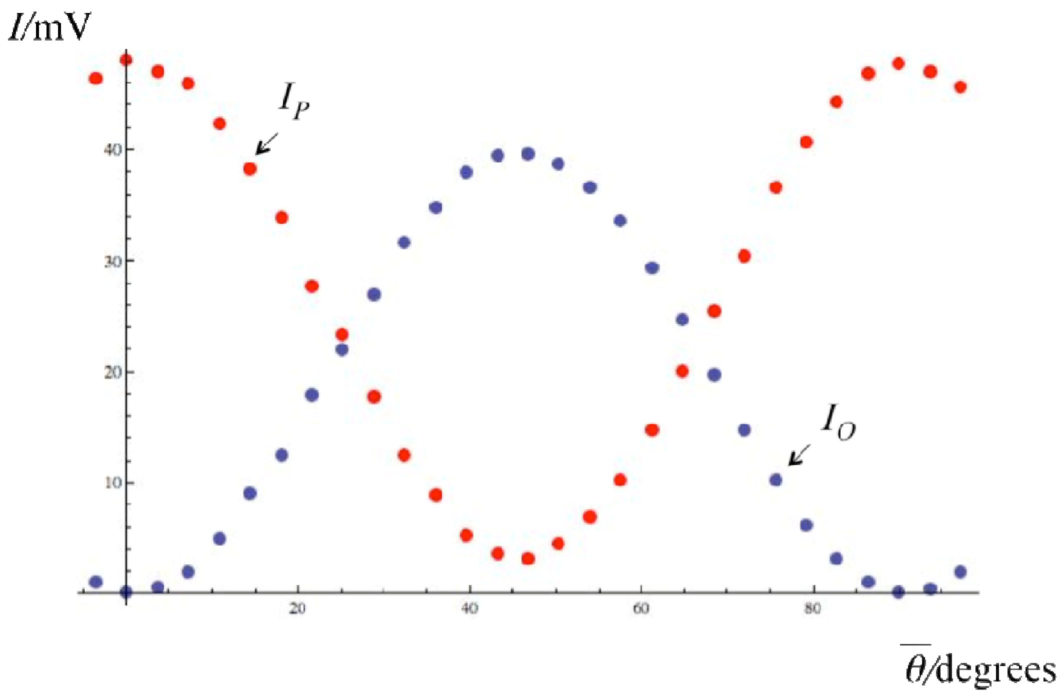
2.2	The angle between two consecutive black lines is $\theta_{\text{int}} = 3.6$ degrees because there are 100 lines.	0.25

Tasks 2.3 Measuring I_P and I_O.Use additional sheets if necessary.

TABLE I (3 points)

$\overline{\theta}$ (degrees)	$(I_P \pm 1)\times 10^{-3}$ V	$(I_O \pm 1)\times 10^{-3}$ V
-3.6	46.4	1.1
0	48.1	0.2
3.6	47.0	0.6
7.2	46.0	2.0
10.8	42.3	4.9
14.4	38.2	9.0
18.0	33.9	12.5

21.6	27.7	17.9
25.2	23.4	22.0
28.8	17.8	27.0
32.4	12.5	31.7
36.0	8.8	34.8
39.6	5.2	38.0
43.2	3.6	39.4
46.8	3.2	39.6
50.4	4.5	38.7
54.0	6.9	36.6
57.6	10.3	33.6
61.2	14.7	29.4
64.8	20.1	24.7
68.4	25.4	19.7
72.0	30.5	14.7
75.6	36.6	10.2
79.2	40.7	6.1
82.8	44.3	3.2
86.4	46.9	1.0
90.0	47.8	0.2
93.6	47.0	0.4
97.2	45.7	2.0



Parallel I_P and perpendicular I_O intensities vs angle $\overline{\theta}$.

GRAPH NOT REQUIRED!

Task 2.4 Finding an appropriate zero for θ.

2.4 *a) Graphical analysis* 1.0

The value for the shift is $\delta\bar{\theta} = -1.0$ degrees.

Add the graph paper with the analysis of this Task.

b) Numerical analysis

From Table I choose the first three points of $\bar{\theta}$ and $I_O(\bar{\theta})$:
(intensities in millivolts)

$(x_1, y_1) = (-3.6, 1.1) \qquad (x_2, y_2) = (0, 0.2) \qquad (x_3, y_3) = (3.6, 0.6)$

We want to fit $y = ax^2 + bx + c$. This gives three equations:

$$1.1 = a(3.6)^2 - b(3.6) + c$$
$$0.2 = c$$
$$0.6 = a(3.6)^2 + b(3.6) + c$$

second in first $\Rightarrow \quad b = \dfrac{-0.9 + a(3.6)^2}{3.6}$

in third $\Rightarrow \quad 0.6 = a\left((3.6)^2 + (3.6)^2\right) - 0.9 + 0.2$

$\Rightarrow \quad a = 0.050 \qquad b = -0.069$

The minimum of the parabola is at:

$$\bar{\theta}_{\min} = -\frac{b}{2a} \approx 0.7 \text{ degrees}$$

Therefore, $\delta\bar{\theta} = -0.7$ degrees.

PINK 40th International Physics Olympiad
Merida, Yucatan, Mexico, July 2009

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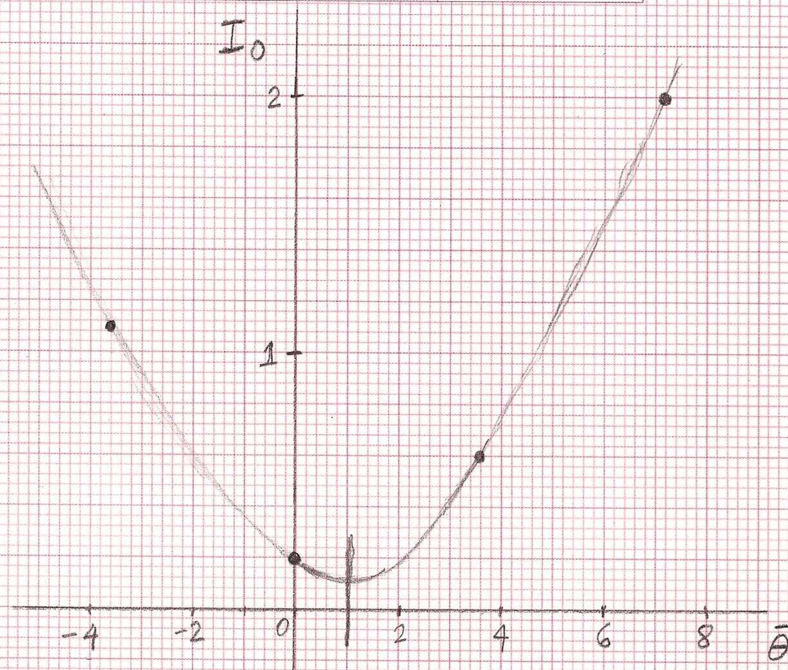


$\bar{\theta}_{min} \approx 1.0$ degrees

4 points

$\bar{\theta}$	(I_0/mV)
−3.6	1.1
0.0	0.2
3.6	0.6
7.2	2.0

Task 2.5 Choosing the appropriate variables.

2.5	Equation (2.4) for the perpendicular intensity is $\bar{I}_O(\theta)=\frac{1}{2}(1-\cos\Delta\phi)\sin^2(2\theta)$ This can be cast as a straight line $y=mx+b$, with $y=\bar{I}_O(\theta)$, $x=\sin^2(2\theta)$ and $m=\frac{1}{2}(1-\cos\Delta\phi)$ from which the phase may be obtained. **NOTE:** This is not the only way to obtain the phase difference. One may, for instance, analyze the 4 maxima of either $\bar{I}_P(\theta)$ or $\bar{I}_O(\theta)$.	0.5

Task 2.6 Statistical analysis and the phase difference.

2.6	To perform the statistical analysis, we shall then use $y=\bar{I}_O(\theta)$ and $x=\sin^2(2\theta)$.	1.0

	Since for $\theta: 0 \rightarrow \frac{\pi}{4}, \quad x: 0 \rightarrow 1$, we use only 12 pairs of data points to cover this range, as given in Table II. x may be left without uncertainty since it is a setting. The uncertainty in y may be calculated as $\Delta \bar{I}_O = \sqrt{\left(\frac{\partial \bar{I}_O}{\partial I_O}\right)^2 \Delta I_O^2 + \left(\frac{\partial \bar{I}_P}{\partial I_P}\right)^2 \Delta I_P^2}$ and one gets $\Delta \bar{I}_O = \frac{\sqrt{I_O^2 + I_P^2}}{(I_O + I_P)^2} \Delta I_O \approx 0.018$, approximately the same for all values.	

TABLE II

$\bar{\theta}$ (degrees)	$x = \sin^2(2\theta)$	$y = \bar{I}_O \pm 0.018$
2.9	0.010	0.013
6.5	0.051	0.042
10.1	0.119	0.104
13.7	0.212	0.191
17.3	0.322	0.269
20.9	0.444	0.392
24.5	0.569	0.484
28.1	0.690	0.603
31.7	0.799	0.717
35.3	0.890	0.798
38.9	0.955	0.880
42.5	0.992	0.916

2.6	We now perform a least square analysis for the variables y vs x in Table II. The slope and y-intercept are: $m \pm \Delta m = 0.913 \pm 0.012$ $b \pm \Delta b = -0.010 \pm 0.008$ The formulas for this analysis are:	1.75

$$m = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} y_{n'}}{\Delta}$$

$$b = \frac{\sum_{n=1}^{N} x_n^2 \sum_{n'=1}^{N} y_{n'} - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} x_{n'} y_{n'}}{\Delta}$$

where

$$\Delta = N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2$$

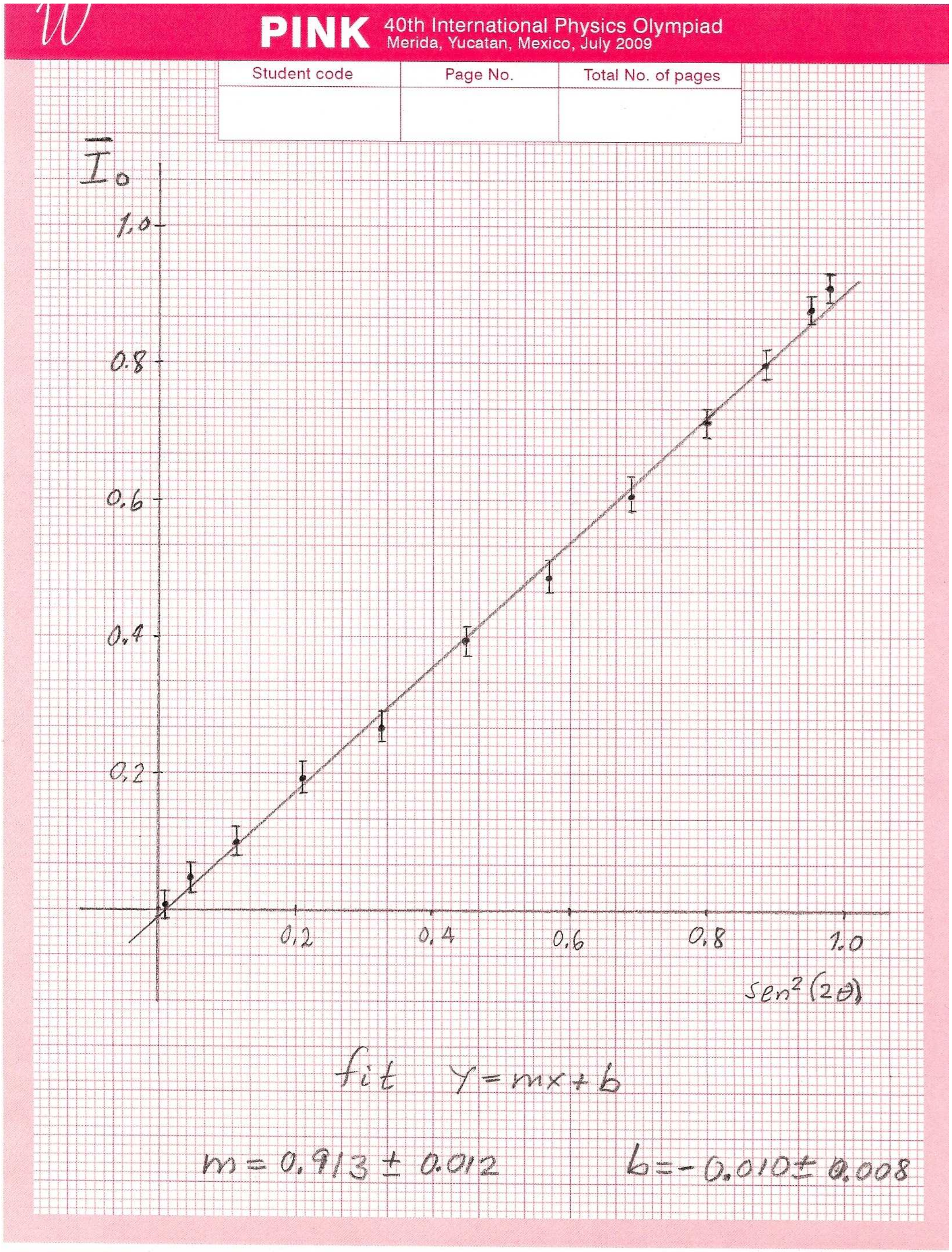
with N the number of data points.
The uncertainty is calculated as

$$(\Delta m)^2 = N\frac{\sigma^2}{\Delta} \quad , \quad (\Delta b)^2 = \frac{\sigma^2}{\Delta}\sum_{n=1}^{N} x_n^2 \qquad \text{with,}$$

$$\sigma^2 = \frac{1}{N-2}\sum_{n=1}^{N}(y_n - b - mx_n)^2$$

with $N = 12$ in this example.

Include the accompanying plot or plots.



2.6	Calculate the value of the phase $\Delta\phi$ in radians in the interval $[0,\pi]$. From the slope $m = \frac{1}{2}(1-\cos\Delta\phi)$, one finds $\Delta\phi \pm \Delta(\Delta\phi) = 2.54 \pm 0.04$ Write down the formulas for the calculation of the uncertainty. We see that,	0.5

	$\Delta m = \left\|\frac{\partial m}{\partial \Delta\phi}\right\| \Delta(\Delta\phi) = \frac{1}{2}\sin(\Delta\phi)\Delta(\Delta\phi)$, therefore, $\Delta(\Delta\phi) = \frac{2\Delta m}{\sin(\Delta\phi)}$.	

Task 2.7 Calculating the birefringence $\|n_1 - n_2\|$.

2.7	Write down the width of the slab of mica you used, $L \pm \Delta L = (100 \pm 1)\times 10^{-6}$ m Write down the wavelength you use, $\lambda \pm \Delta\lambda = (663 \pm 25)\times 10^{-9}$ m (from Problem 1) Calculate the birefringence $\|n_1 - n_2\| \pm \Delta\|n_1 - n_2\| = (3.94 \pm 0.16)\times 10^{-3}$ The birefringence is between 0.003 and 0.005. Nominal value 0.004 Write down the formulas you used for the calculation of the uncertainty of the birefringence. Since the width L > 82 micrometers, we use $2\pi - \Delta\phi = \frac{2\pi L}{\lambda}\|n_1 - n_2\|$ The error is $\Delta\|n_1 - n_2\| = \sqrt{\left(\frac{\partial\|n_1 - n_2\|}{\partial\lambda}\right)^2 \Delta\lambda^2 + \left(\frac{\partial\|n_1 - n_2\|}{\partial L}\right)^2 \Delta L^2 + \left(\frac{\partial\|n_1 - n_2\|}{\partial\Delta\phi}\right)^2 \Delta(\Delta\phi)^2}$ $\Delta\|n_1 - n_2\| = \sqrt{\left(\frac{\|n_1 - n_2\|}{\lambda}\right)^2 \Delta\lambda^2 + \left(\frac{\|n_1 - n_2\|}{L}\right)^2 \Delta L^2 + \left(\frac{\lambda}{2\pi L}\right)^2 \Delta(\Delta\phi)^2}$	1.0

	Since the data may appear somewhat disperse and/or the errors in the intensities may be large, a graphical analysis may be performed. In the accompanying plot, it is exemplified a simple graphical analysis: first the main slope is found, then, using the largest deviations one can find two extreme slopes. The final result is, $m = 0.91 \pm 0.08$ and $b = -0.01 \pm 0.04$ The calculation of the birefringence and its uncertainty follows as before. One now finds, $\|n_1 - n_2\| \pm \Delta\|n_1 - n_2\| = (3.94 \pm 0.45) \times 10^{-3}$. A larger (more realistic) error.	

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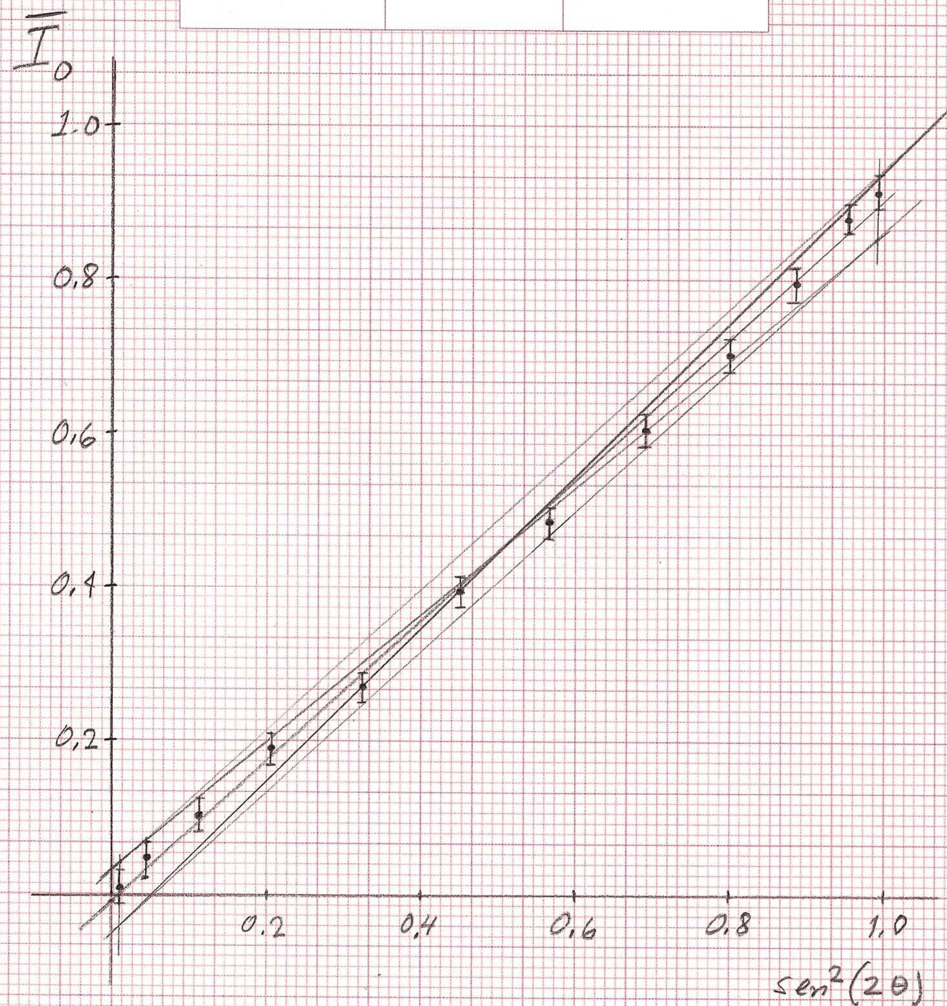
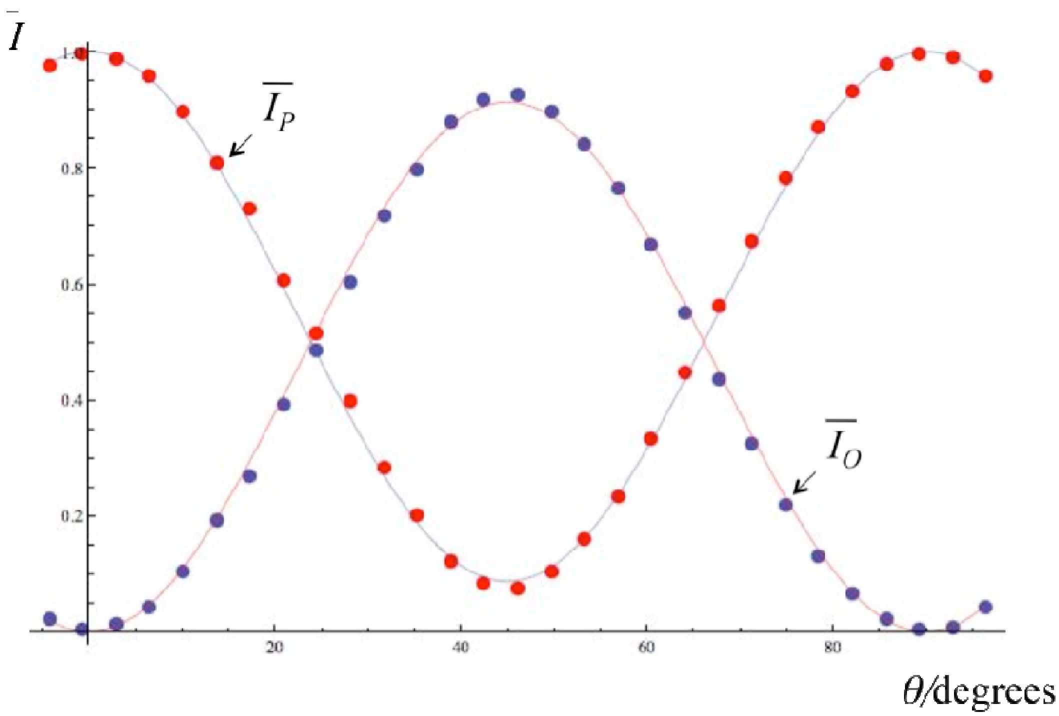


Graphical analysis

$m = 0.91 \pm 0.08$ $b = -0.01 \pm 0.04$



Comparison of experimental data (normalized intensities $\bar{I}_P$ and $\bar{I}_O$) with fitting (equations (2.3) and (2.4)) using the calculated value of the phase difference $\Delta\phi$.

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